Artificial Intelligence & Algorithms

UCLouvain

Anytime Weighted Model Counting with Approximation Guarantees For Probabilistic Inference

Alexandre Dubray, Pierre Schaus, Siegfried Nijssen

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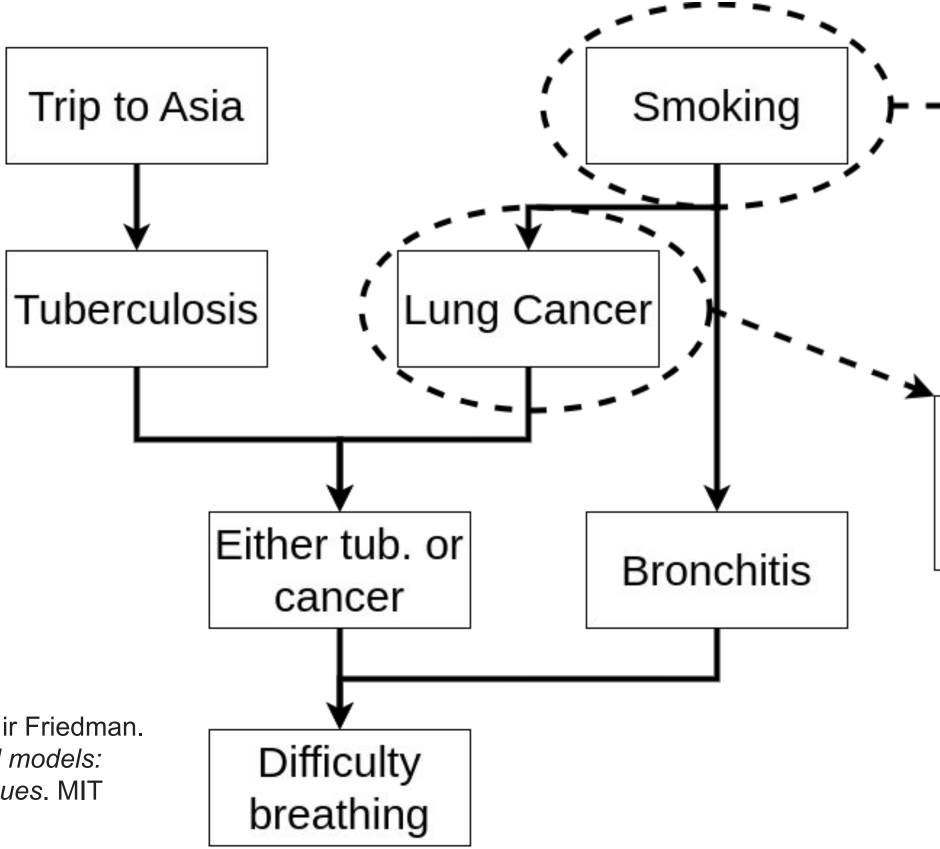
Probabilistic Inference: Bayesian Networks

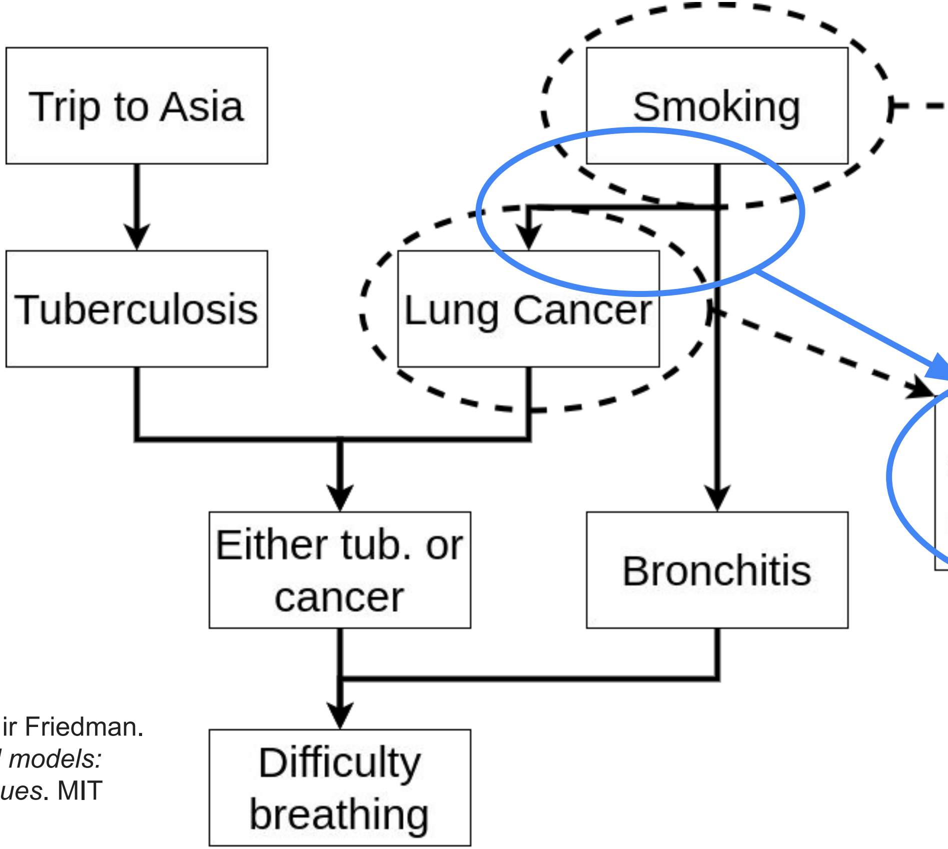


Smoking		
yes	*smt*	*no*
0.1	0.3	0.6

		Lung Cancer	
		yes	*no*
Smoking	*yes*	0.9	0.1
	smt	0.6	0.4
	no	0.2	0.8

Koller, Daphne, and Nir Friedman. *Probabilistic graphical models: principles and techniques*. MIT press, 2009.

Probabilistic Inference: Bayesian Networks



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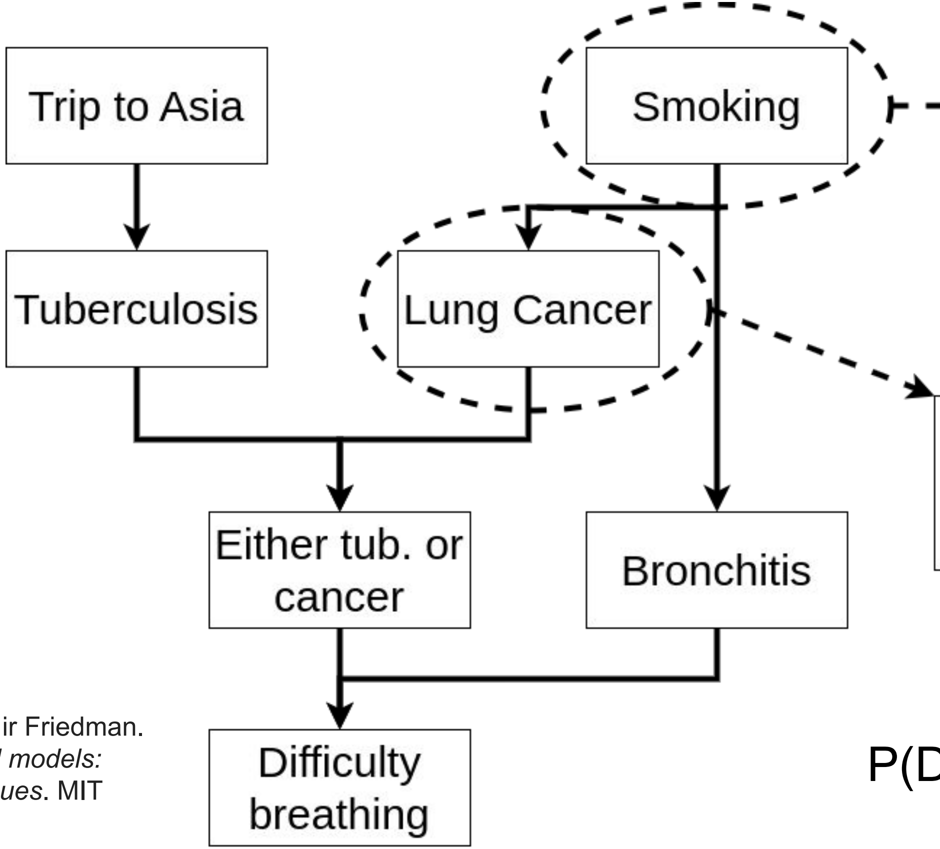
Probabilistic Inference: Query on Bayesian Networks



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P(Difficulty breathing) = ?

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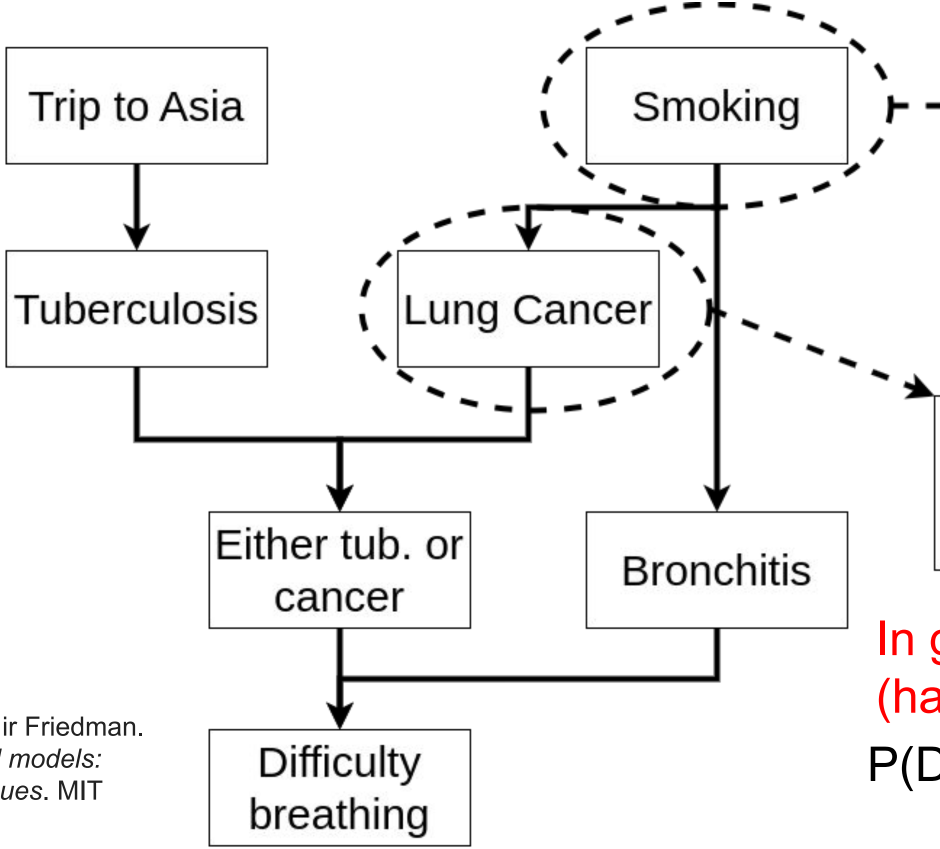
Probabilistic Inference: Query on Bayesian Networks



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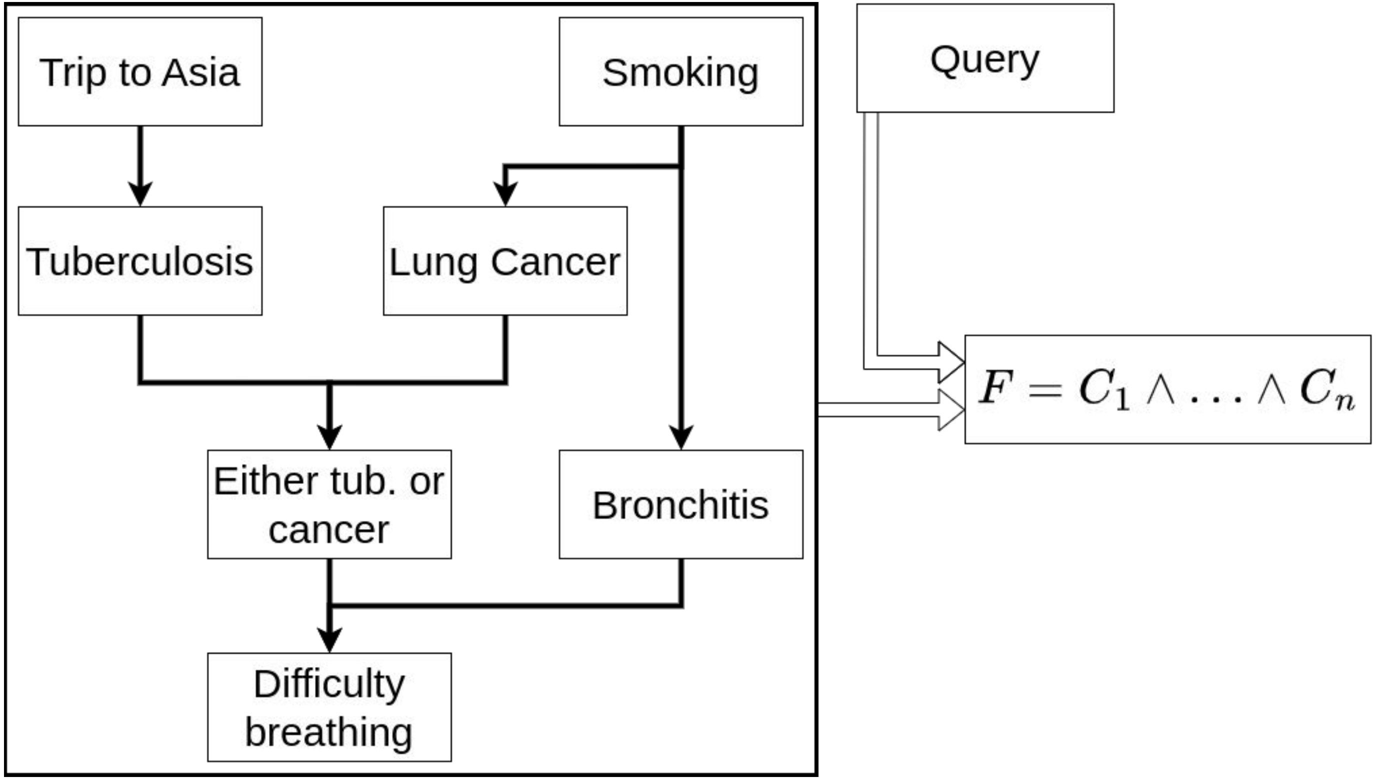
In general, #P-hard problem (harder than NP)

P(Difficulty breathing) = ?

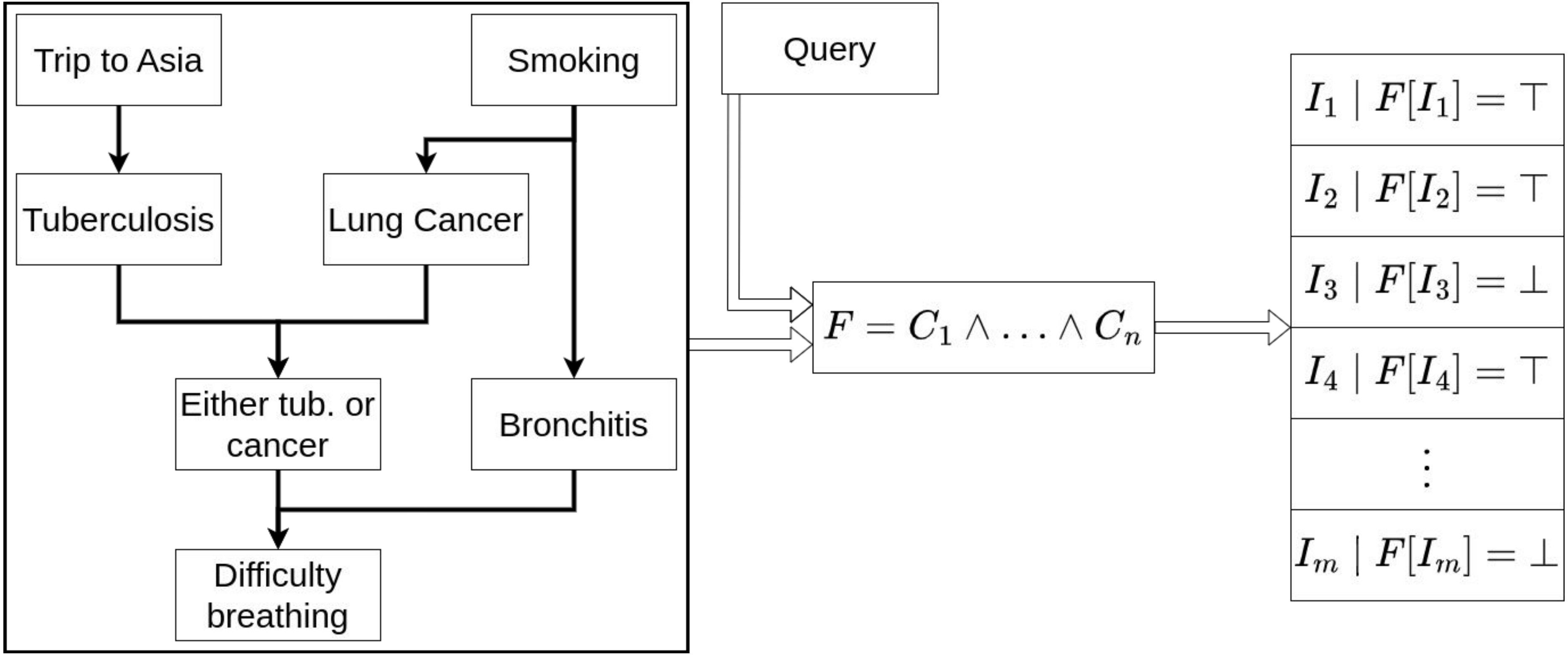
Koller, Daphne, and Nir Friedman. *Probabilistic graphical models: principles and techniques*. MIT press, 2009.

Probabilistic Inference: A Counting Problem



Chavira, Mark, and Adnan Darwiche. "On probabilistic inference by weighted model counting." *Artificial Intelligence* 172.6-7 (2008): 772-799.

Probabilistic Inference: A Counting Problem



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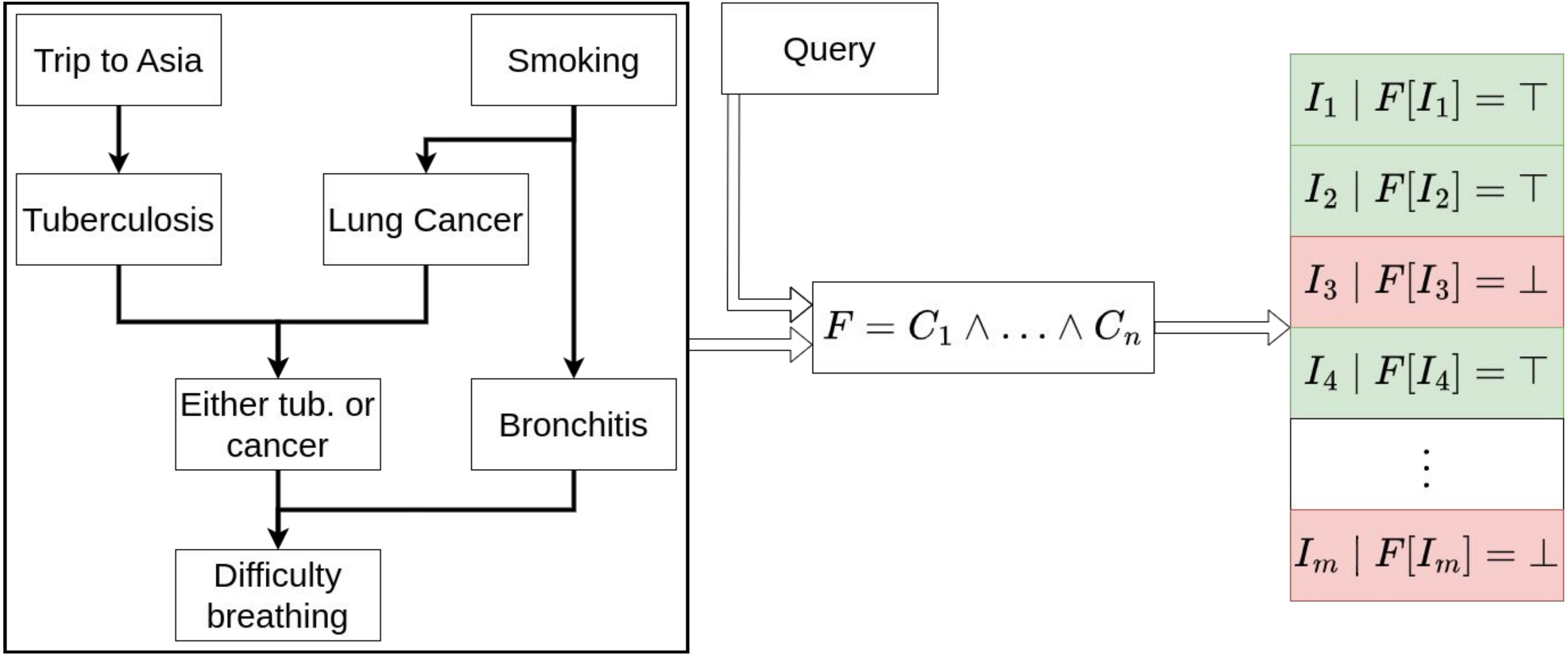

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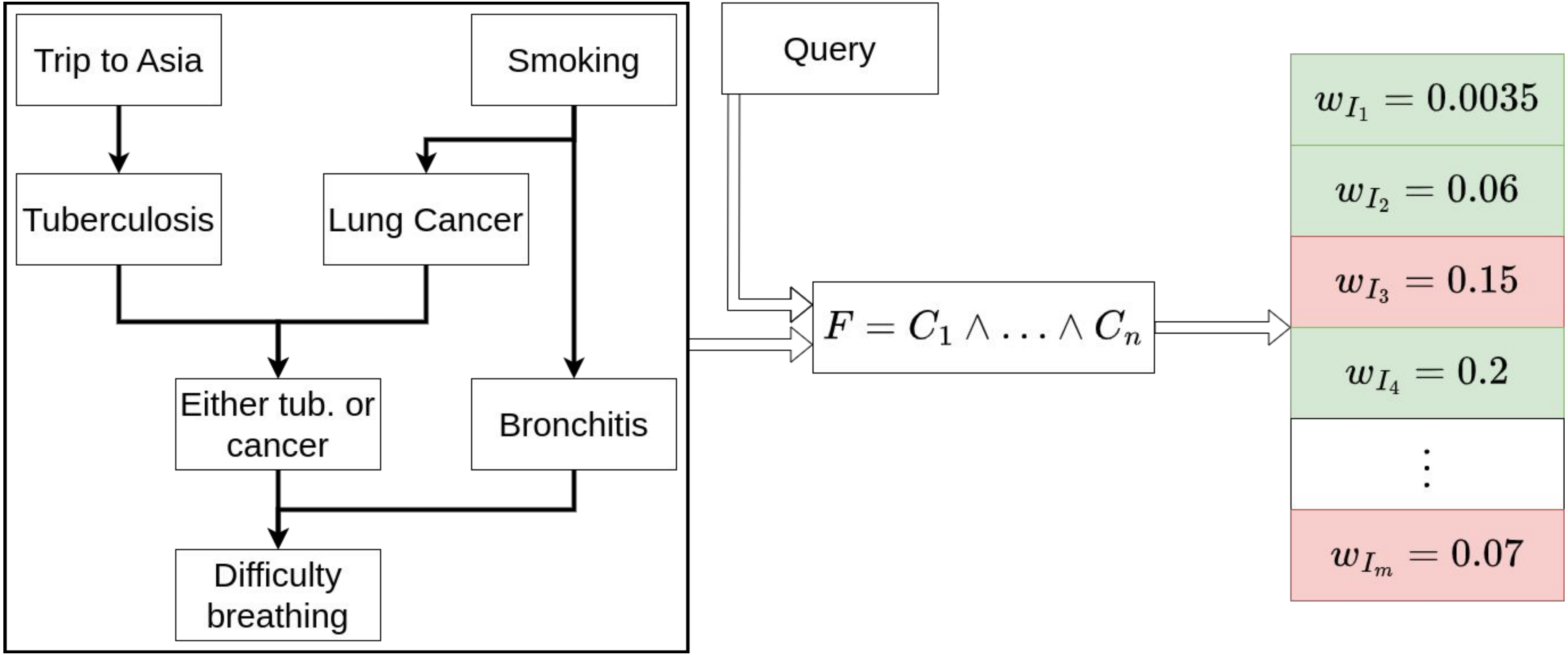

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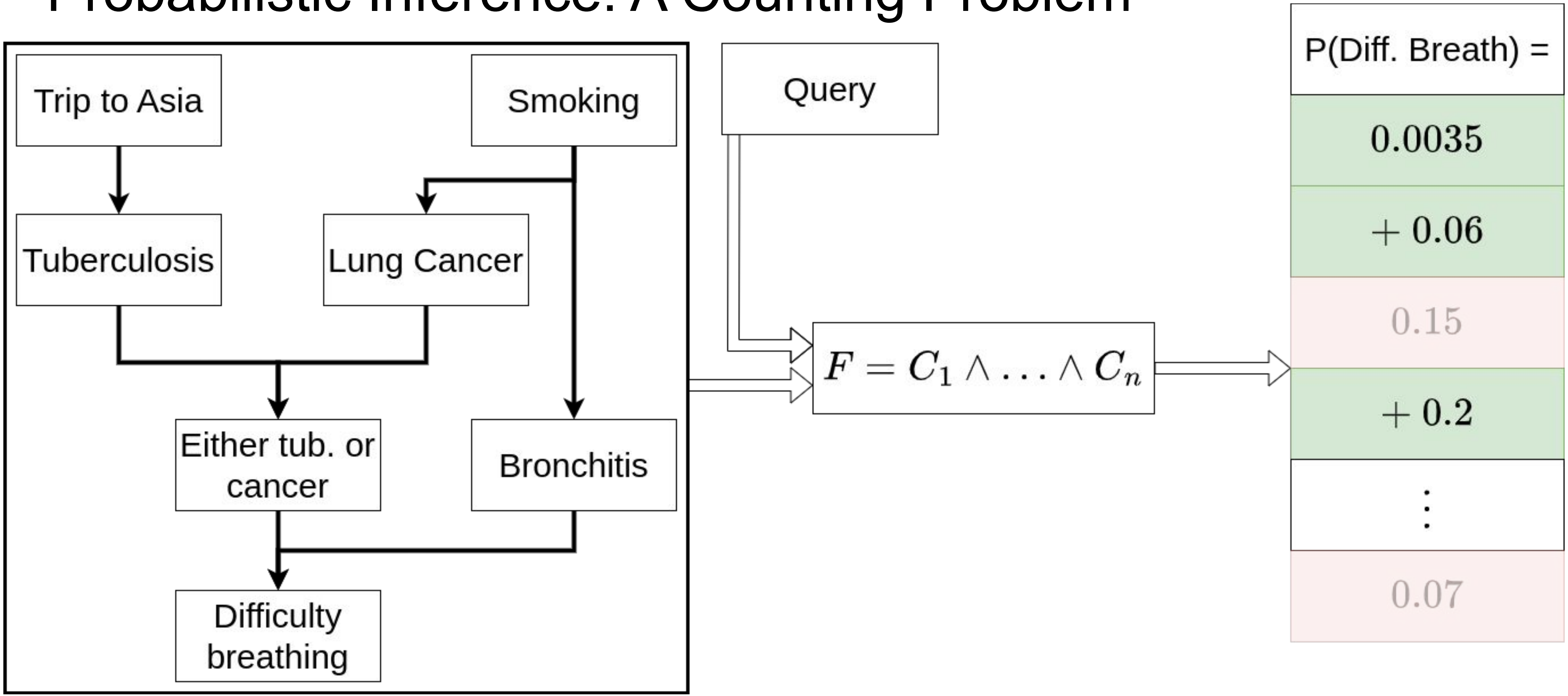
Probabilistic Inference: A Counting Problem



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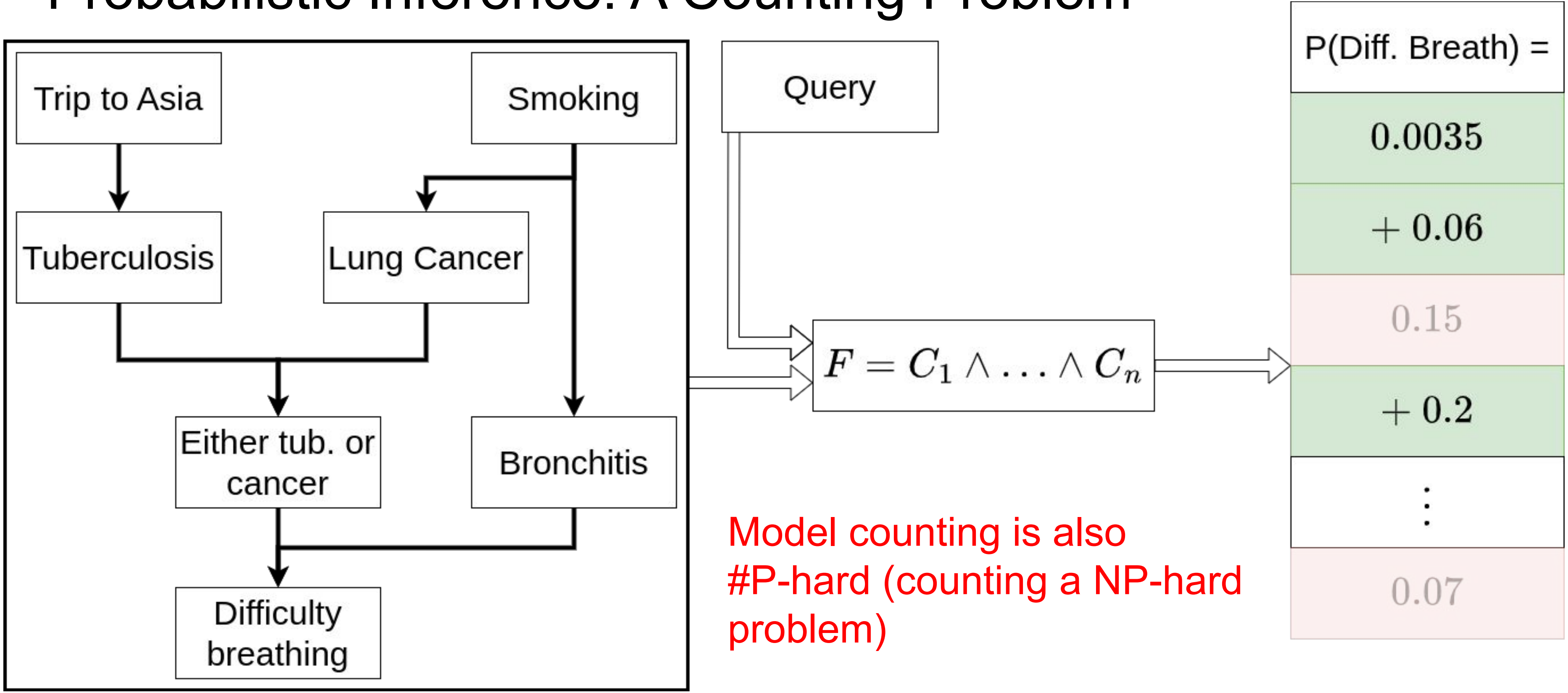
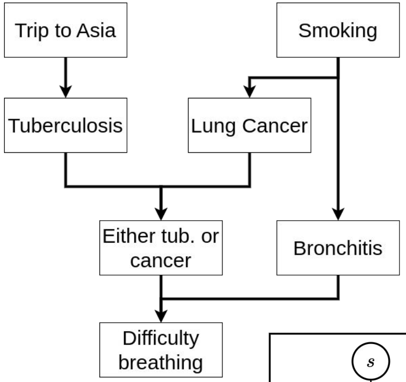
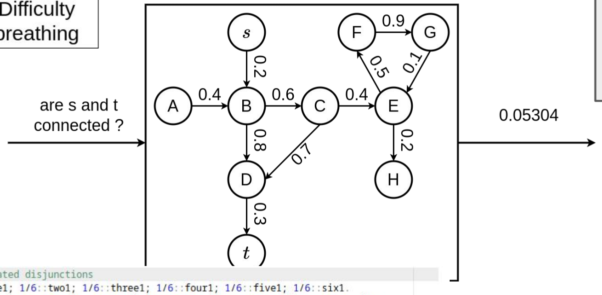


Model counting is also #P-hard (counting a NP-hard problem)

Chavira, Mark, and Adnan Darwiche. "On probabilistic inference by weighted model counting." *Artificial Intelligence* 172.6-7 (2008): 772-799.
Valiant, Leslie G. "The complexity of enumeration and reliability problems." *siam Journal on Computing* 8.3 (1979): 410-421.

Why Model Counting?





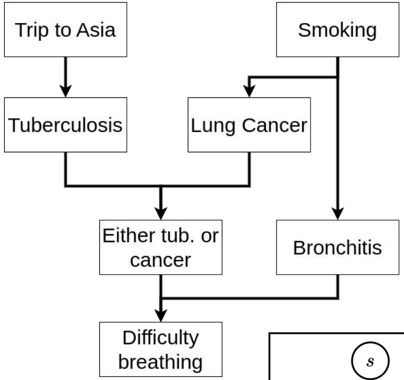

```
% annotated disjunctions
1/6::one1; 1/6::two1; 1/6::three1; 1/6::four1; 1/6::five1; 1/6::six1.
0.15::one2; 0.15::two2; 0.15::three2; 0.15::four2; 0.15::five2; 0.25::six2.

% Rules:
twoSix :- six1, six2.

someSix :- six1.
someSix :- six2.

% Queries:
query(six1).
query(six2).
query(twoSix).
query(someSix).
```

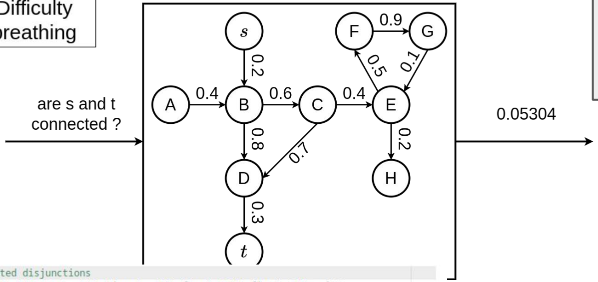
Evaluate

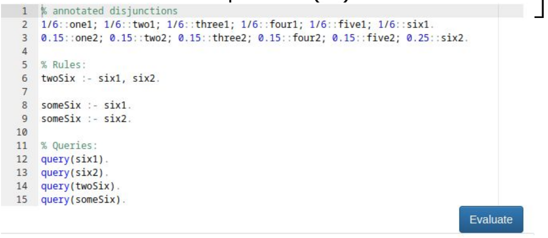
Model counting

Probability

Why Model Counting? *“One solver to solve them all”*







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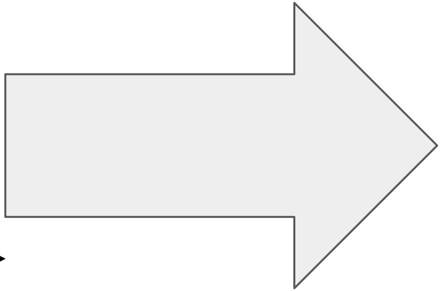
Model counting



Probability

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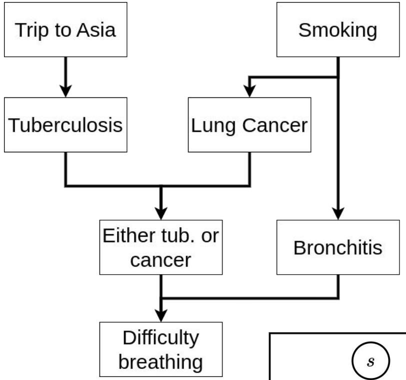

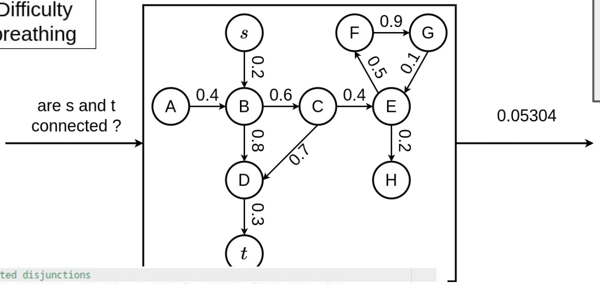


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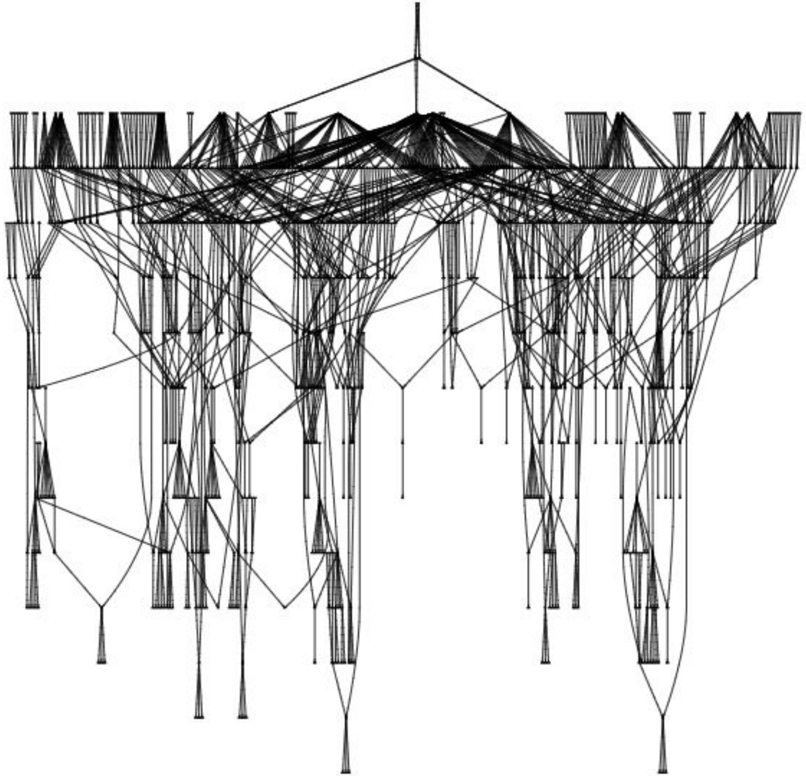
Evaluate

Model counting

Probability

Probabilistic Inference for Large ~~language~~ Models



Computing exact probabilities

"That's a no-no" - Donald Trump

Computing **approximate** probabilities

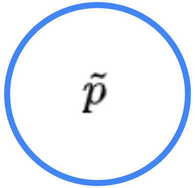
"Yes we can" - Barack Obama

Approximate Inference: Bounding the Error

$$\tilde{p}$$

Roth, Dan. "On the hardness of approximate reasoning." *Artificial Intelligence* 82.1-2 (1996): 273-302.

Approximate Inference: Bounding the Error



Approximated probability

Roth, Dan. "On the hardness of approximate reasoning." *Artificial Intelligence* 82.1-2 (1996): 273-302.

Approximate Inference: Bounding the Error

$$\frac{p^\star}{1+\varepsilon} \leq \quad \tilde{p}$$

Roth, Dan. "On the hardness of approximate reasoning." *Artificial Intelligence* 82.1-2 (1996): 273-302.

Approximate Inference: Bounding the Error

True probability

$$\frac{p^\star}{1+\varepsilon} \leq \tilde{p}$$

Roth, Dan. "On the hardness of approximate reasoning." *Artificial Intelligence* 82.1-2 (1996): 273-302.

Approximate Inference: Bounding the Error

$$\frac{p^\star}{1+\varepsilon} \leq \tilde{p}$$

$$\Updownarrow$$

$$p^\star \leq \tilde{p}(1+\varepsilon)$$

Upper bound on the true probability !

Roth, Dan. "On the hardness of approximate reasoning." *Artificial Intelligence* 82.1-2 (1996): 273-302.

Approximate Inference: Bounding the Error

Epsilon guarantee bounds the true probability with relative error

$$\frac{p^\star}{1+\varepsilon} \leq \quad \tilde{p} \ \leq (1+\varepsilon)p^\star$$

Roth, Dan. "On the hardness of approximate reasoning." *Artificial Intelligence* 82.1-2 (1996): 273-302.

Approximate Inference: Bounding the Error

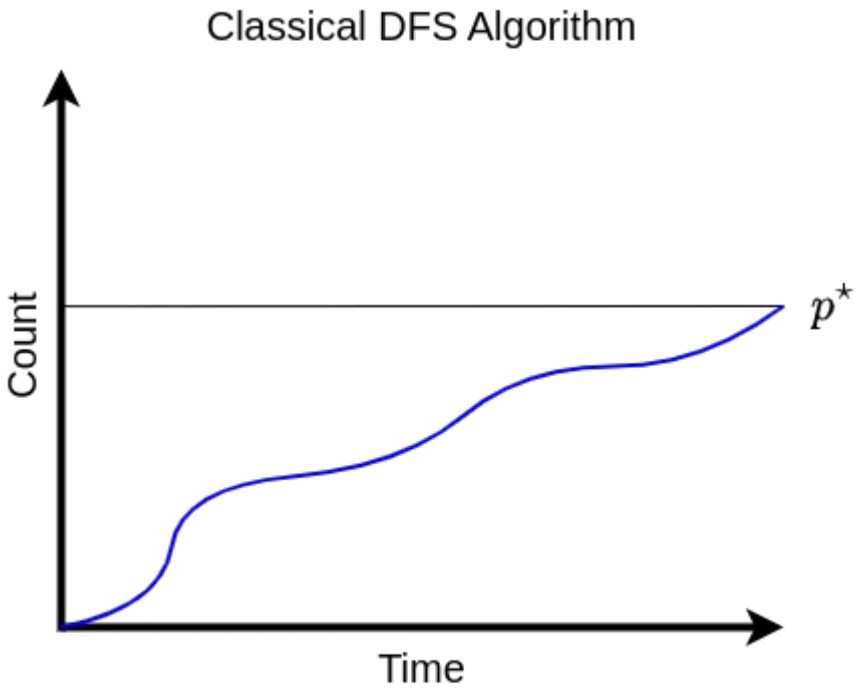
$$\frac{p^\star}{1+\varepsilon} \leq \quad \tilde{p} \leq (1+\varepsilon)p^\star$$

$$P\left[\frac{p^\star}{1+\varepsilon} \leq \tilde{p} \leq (1+\varepsilon)p^\star\right] \geq 1-\delta$$

Epsilon-delta guarantee bounds the true count probabilistically

Roth, Dan. "On the hardness of approximate reasoning." *Artificial Intelligence* 82.1-2 (1996): 273-302.

Anytime Algorithms: Expectations



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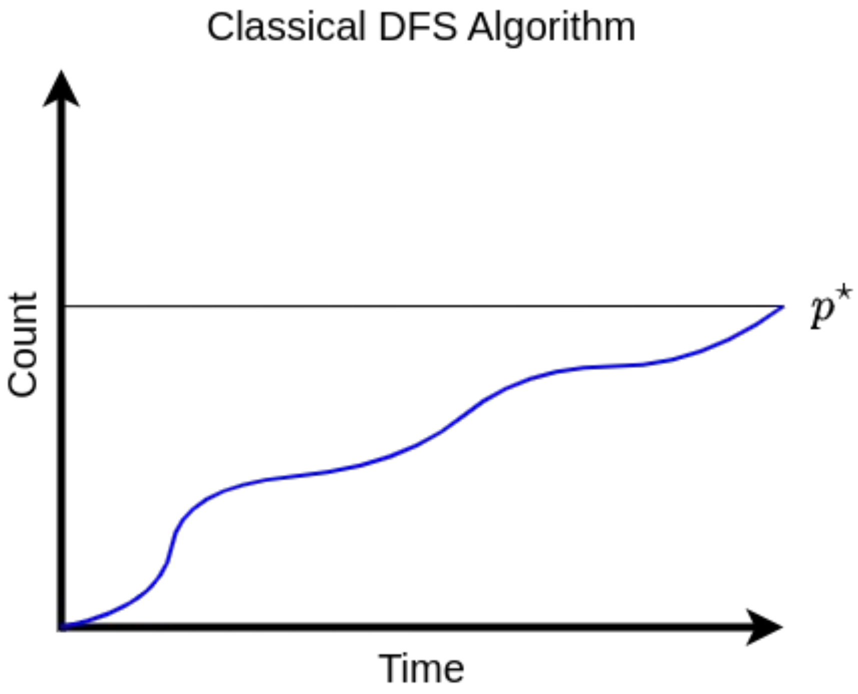

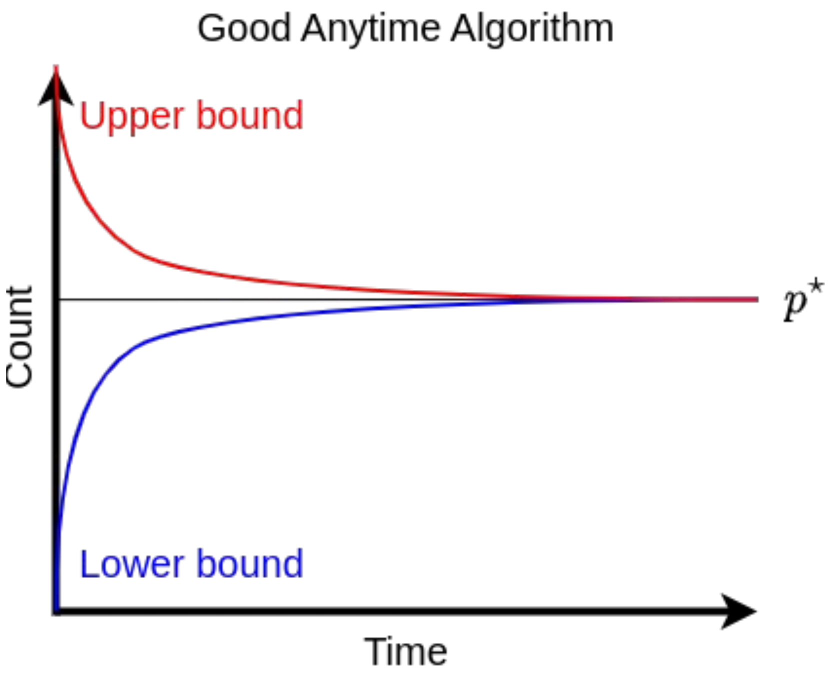

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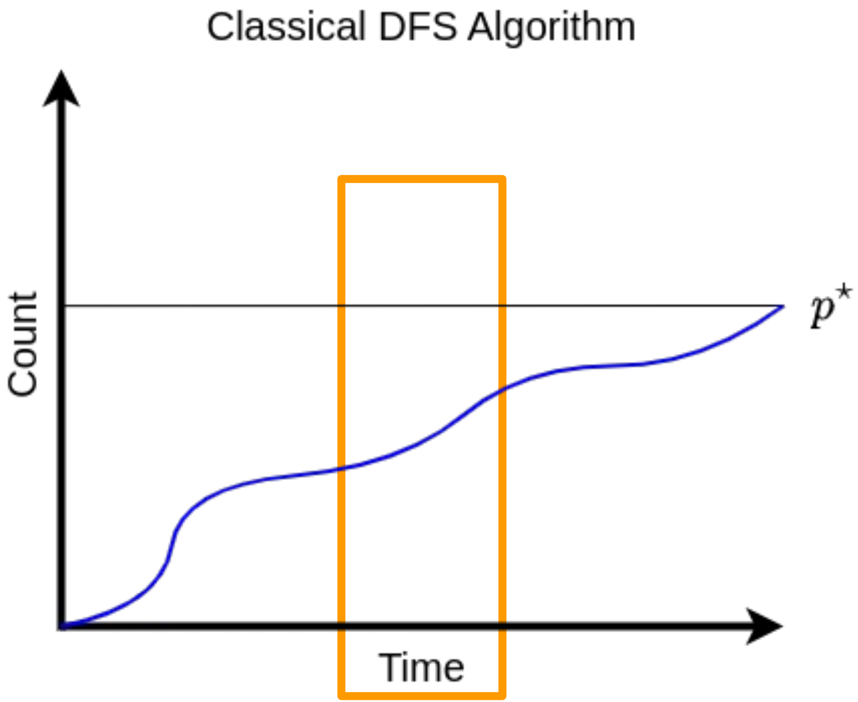

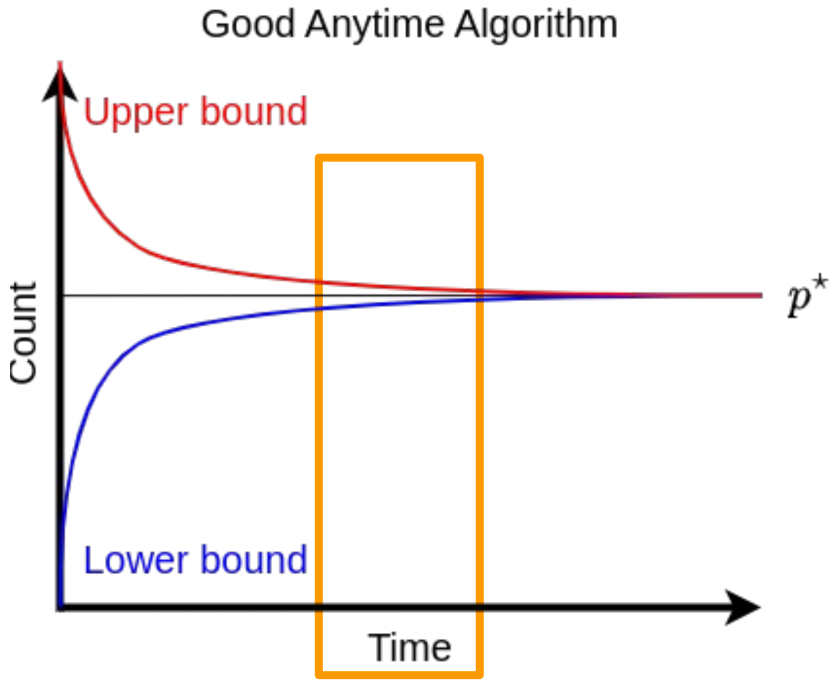

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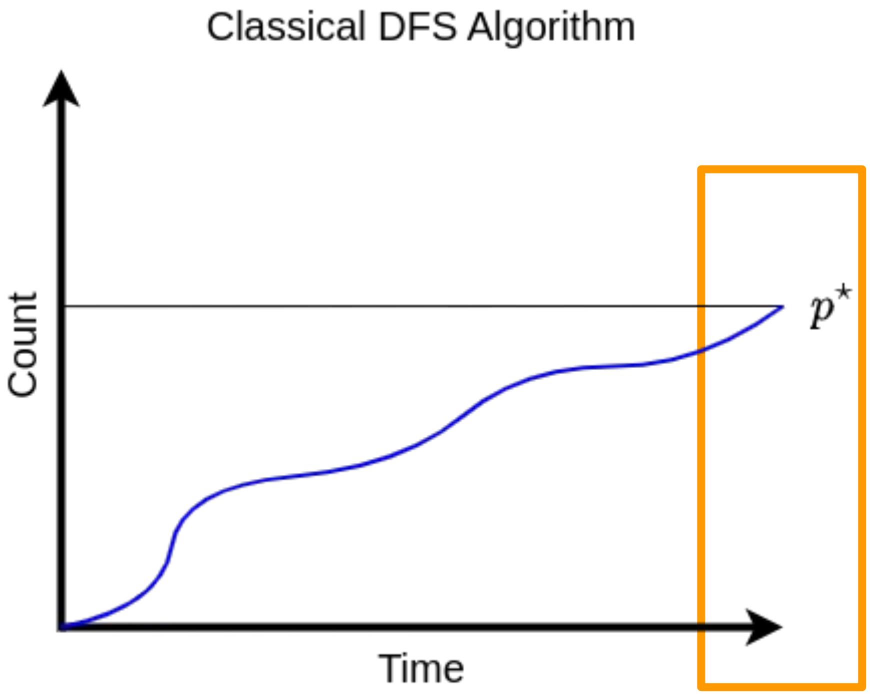

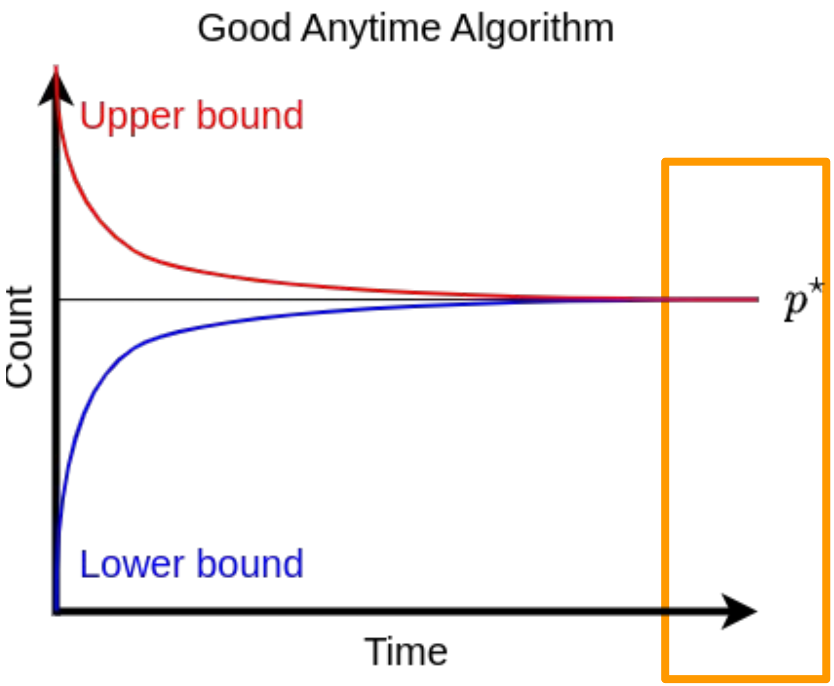

Fifty shades of Approximate Inference

	Anytime	Guarantees	Lower bound	Upper bound
d4/gpmc/ exactMC/…	❌	Exact on completion	❌	❌

Lagniez, Jean-Marie, and Pierre Marquis. "A recursive algorithm for projected model counting." *Proceedings of the AAAI Conference on Artificial Intelligence*. Vol. 33. No. 01. 2019.

Lagniez, Jean-Marie, and Pierre Marquis. "An Improved Decision-DNNF Compiler." *IJCAI*. Vol. 17. 2017.

Ryosuke Suzuki, Kenji Hashimoto, and Masahiko Sakai. Improvement of projected model counting solver with component decomposition using SAT solving in components.

Yong Lai, Kuldeep S. Meel, and Roland HC Yap. The power of literal equivalence in model counting. In Proceedings of the AAAI Conference on Artificial Intelligence, volume 35, 2021.

Fifty shades of Approximate Inference

	Anytime	Guarantees	Lower bound	Upper bound
d4/gpmc/ exactMC/…	✗	Exact on completion	✗	✗
ApproxMC/ WeightMC	✓	(ε, δ)	✓	✓

Supratik Chakraborty, Kuldeep S. Meel, and Moshe Y. Vardi. Algorithmic improvements in approximate counting for probabilistic inference: From linear to logarithmic SAT calls. In IJCAI, 2016.
Mate Soos, Stephan Gocht, and Kuldeep S. Meel. Tinted, Detached, and Lazy CNF-XOR solving and its Applications to Counting and Sampling. In CCAV, 2020.
Mate Soos and Kuldeep S. Meel. BIRD: Engineering an efficient CNF-XOR SAT solver and its applications to approximate model counting. In Proceedings of the AAAI Conference on Artificial Intelligence
Supratik Chakraborty, Daniel Fremont, Kuldeep Meel, Sanjit Seshia, and Moshe Vardi. Distribution-aware sampling and weighted model counting for SAT. In AAAI, 2014.

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PartialKC / SampleSAT	✓	Probability on the LB	✓	✓

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Toulbar	✓	$(\varepsilon, 0)$	✓	✓

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Toulbar	✓	$(\varepsilon, 0)$	✓	✓
Our contribution	✓	$(\varepsilon, 0)$	✓	✓

Fifty shades of Approximate Inference

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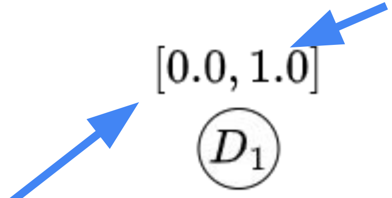
Schlandals	✓	$(\varepsilon, 0)$	✓	✓

DFS with Bounds

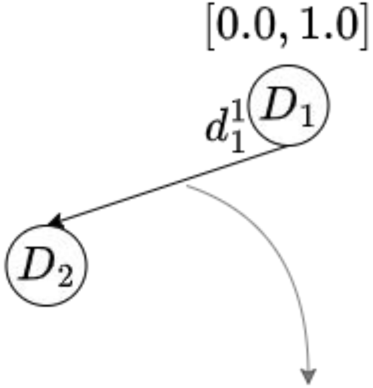
Upper bound on the (sub-)problem



Lower bound on the (sub-)problem

DFS with Bounds

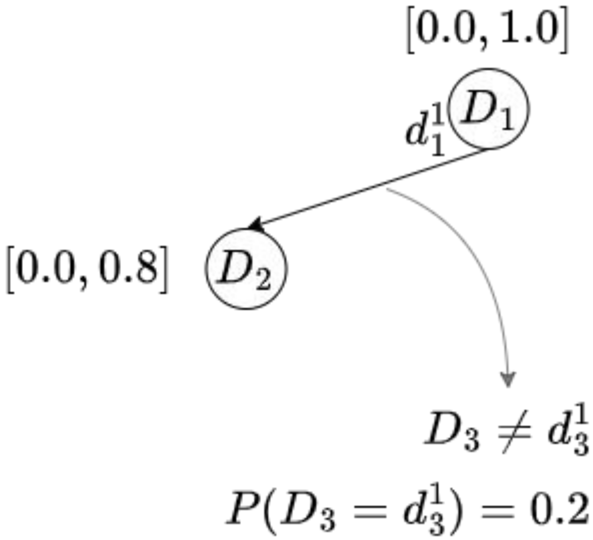
$D_3 = \{d_3^1, d_3^2, d_3^3, d_3^4\}$



$D_3 \neq d_3^1$

$P(D_3 = d_3^1) = 0.2$

DFS with Bounds



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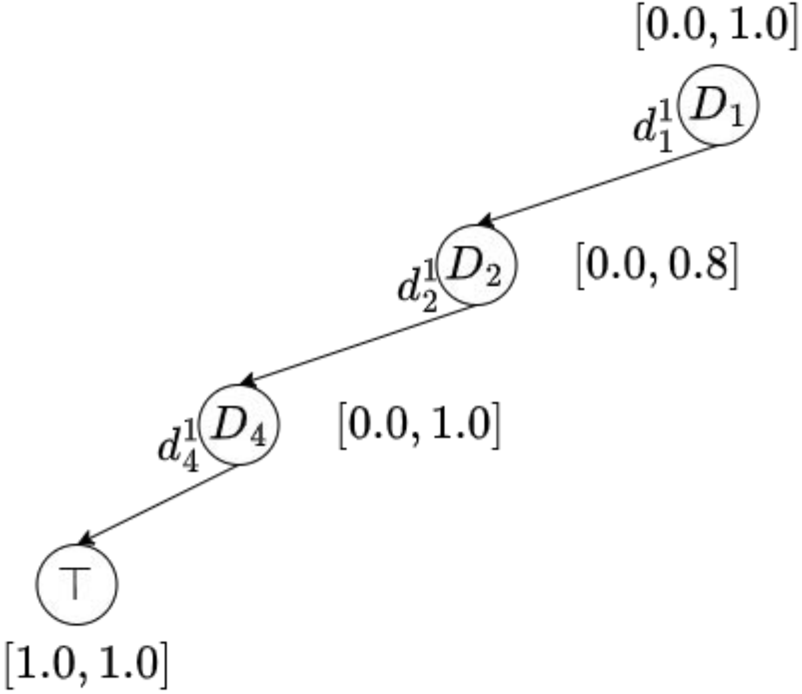

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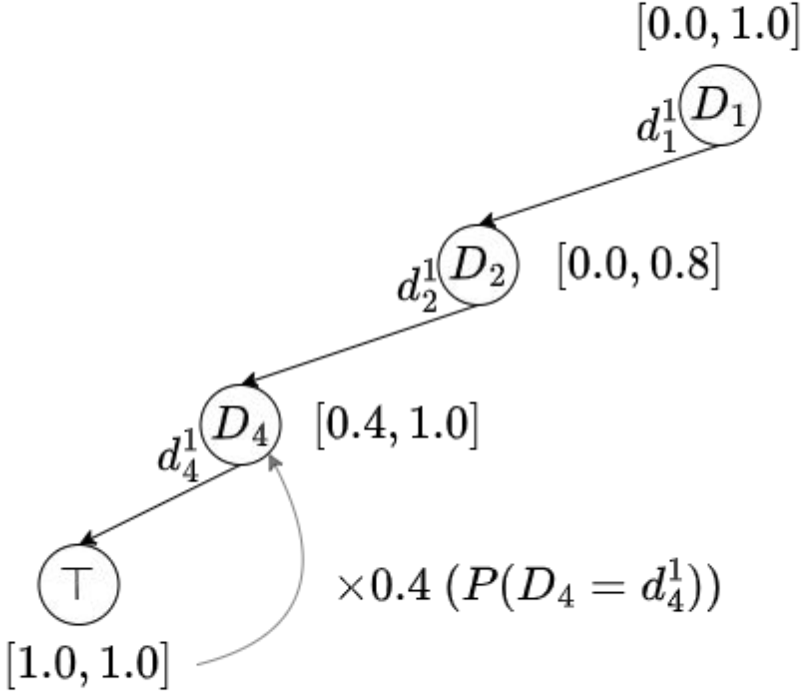

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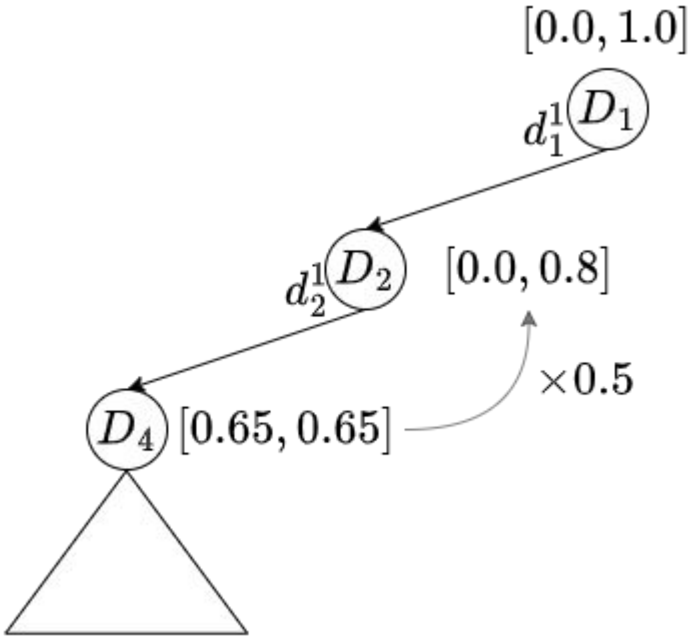

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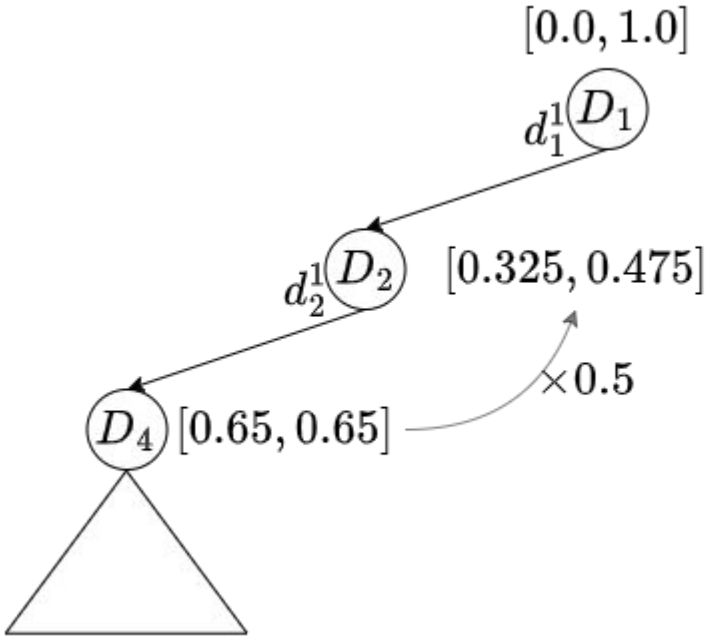

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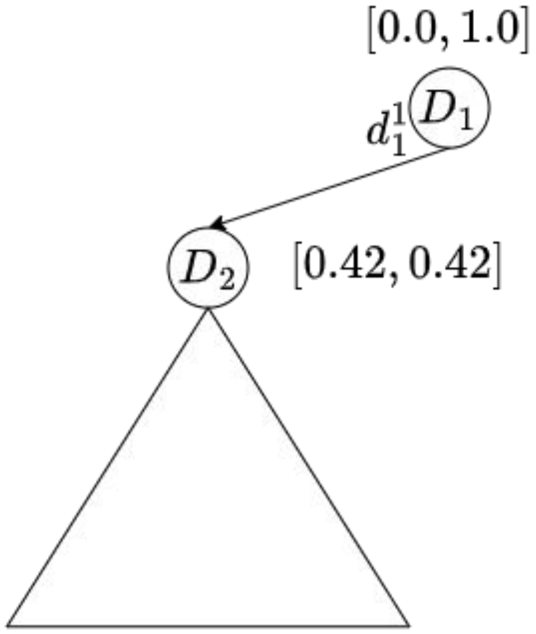

Fifty shades of Approximate Inference

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From bounds to epsilon-guarantees

$$\frac{p^\star}{1+\varepsilon} \leq \tilde{p} \leq (1+\varepsilon)p^\star$$

From bounds to epsilon-guarantees

$$\frac{p^\star}{1+\varepsilon} \leq \tilde{p} \leq (1+\varepsilon)p^\star$$

?

$$f(lb, ub)$$

From bounds to epsilon-guarantees

$$\frac{p^\star}{1+\varepsilon} \leq \tilde{p} \leq (1+\varepsilon)p^\star$$

?

$$f(lb, ub) \qquad ub \leq lb(1+\varepsilon)^2$$

From bounds to epsilon-guarantees

$$\frac{p^\star}{1+\varepsilon} \leq \tilde{p} \leq (1+\varepsilon)p^\star$$

?

$$f(lb, ub)$$

$$ub \leq lb(1+\varepsilon)^2 \Rightarrow \frac{p^\star}{1+\varepsilon} \leq \sqrt{lb \times ub} \leq (1+\varepsilon)p^\star$$

From bounds to epsilon-guarantees

$$\frac{p^\star}{1+\varepsilon} \leq \tilde{p} \leq (1+\varepsilon)p^\star$$

?

$$f(lb, ub)$$

Proof in the paper !

$$ub \leq lb(1+\varepsilon)^2 \Rightarrow \frac{p^\star}{1+\varepsilon} \leq \sqrt{lb \times ub} \leq (1+\varepsilon)p^\star$$

From bounds to epsilon-guarantees

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$$\varepsilon_{lb,ub} = \sqrt{\frac{ub}{lb}} - 1$$

Lower- and upper-bounds induced a minimal approximation factor !

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Ordering Matters (as usual)

Weight	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$	0.0035	1.0
$w_{I_2} = 0.06$		
$w_{I_3} = 0.15$		
$w_{I_4} = 0.2$		
$w_{I_5} = 0.07$		
$w_{I_6} = 0.1165$		
$w_{I_7} = 0.4$		

Ordering Matters (as usual)

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$		
$w_{I_2} = 0.06$	0.0635	1.0
$w_{I_3} = 0.15$		
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$w_{I_6} = 0.1165$		
$w_{I_7} = 0.4$		

Ordering Matters (as usual)

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$		
$w_{I_2} = 0.06$		
$w_{I_3} = 0.15$	0.0635	$1 - 0.15 = 0.85$
$w_{I_4} = 0.2$		
$w_{I_5} = 0.07$		
$w_{I_6} = 0.1165$		
$w_{I_7} = 0.4$		

Ordering Matters (as usual)

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$		
$w_{I_2} = 0.06$		
$w_{I_3} = 0.15$		
$w_{I_4} = 0.2$	0.2635	$1 - 0.15 = 0.85$
$w_{I_5} = 0.07$		
$w_{I_6} = 0.1165$		
$w_{I_7} = 0.4$		

Ordering Matters (as usual)

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$		
$w_{I_2} = 0.06$		
$w_{I_3} = 0.15$		
$w_{I_4} = 0.2$		
$w_{I_5} = 0.07$		
$w_{I_6} = 0.1165$		
$w_{I_7} = 0.4$		
	0.2635	$1 - 0.7365 = 0.2635$

Ordering Matters (as usual)

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$	0.0035	1.0
$w_{I_2} = 0.06$		
$w_{I_3} = 0.15$		
$w_{I_4} = 0.2$		
$w_{I_5} = 0.07$		
$w_{I_6} = 0.1165$		
$w_{I_7} = 0.4$		

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_7} = 0.4$	0.0	0.6
$w_{I_4} = 0.2$		
$w_{I_3} = 0.15$		
$w_{I_6} = 0.1165$		
$w_{I_5} = 0.07$		
$w_{I_2} = 0.06$		
$w_{I_1} = 0.0035$		

Ordering Matters (as usual)

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$		
$w_{I_2} = 0.06$	0.0635	1.0
$w_{I_3} = 0.15$		
$w_{I_4} = 0.2$		
$w_{I_5} = 0.07$		
$w_{I_6} = 0.1165$		
$w_{I_7} = 0.4$		

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_7} = 0.4$		
$w_{I_4} = 0.2$	0.2	0.6
$w_{I_3} = 0.15$		
$w_{I_6} = 0.1165$		
$w_{I_5} = 0.07$		
$w_{I_2} = 0.06$		
$w_{I_1} = 0.0035$		

Ordering Matters (as usual)

	$P(query)$	$P(query) = 1 - P(\neg query)$		$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$			$w_{I_7} = 0.4$		
$w_{I_2} = 0.06$			$w_{I_4} = 0.2$		
$w_{I_3} = 0.15$	0.0635	$1 - 0.15 = 0.85$	$w_{I_3} = 0.15$	0.2	0.45
$w_{I_4} = 0.2$			$w_{I_6} = 0.1165$		
$w_{I_5} = 0.07$			$w_{I_5} = 0.07$		
$w_{I_6} = 0.1165$			$w_{I_2} = 0.06$		
$w_{I_7} = 0.4$			$w_{I_1} = 0.0035$		

Ordering Matters (as usual)

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_1} = 0.0035$		
$w_{I_2} = 0.06$		
$w_{I_3} = 0.15$		
$w_{I_4} = 0.2$	0.2635	$1 - 0.15 = 0.85$
$w_{I_5} = 0.07$		
$w_{I_6} = 0.1165$		
$w_{I_7} = 0.4$		

	$P(query)$	$P(query) = 1 - P(\neg query)$
$w_{I_7} = 0.4$		
$w_{I_4} = 0.2$		
$w_{I_3} = 0.15$		
$w_{I_6} = 0.1165$	0.2	0.3335
$w_{I_5} = 0.07$		
$w_{I_2} = 0.06$		
$w_{I_1} = 0.0035$		

Ordering Matters (as usual)

$P(query)$

$w_{I_1} = 0.0035$

$w_{I_2} = 0.06$

$w_{I_3} = 0.15$

$w_{I_4} = 0.2$

$w_{I_5} = 0.07$

$w_{I_6} = 0.1165$

$w_{I_7} = 0.4$

0.2635

$P(query) = 1 - P(\neg query)$

$1 - 0.7365 = 0.2635$

$P(query)$

$w_{I_7} = 0.4$

$w_{I_4} = 0.2$

$w_{I_3} = 0.15$

$w_{I_6} = 0.1165$

$w_{I_5} = 0.07$

$w_{I_2} = 0.06$

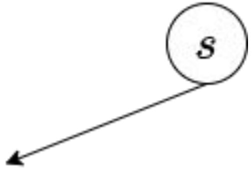
$w_{I_1} = 0.0035$

0.2635

$P(query) = 1 - P(\neg query)$

0.2635

Finding Most Likely Interpretations First



Which value to choose first ?
Yes, no, or Sometimes ?

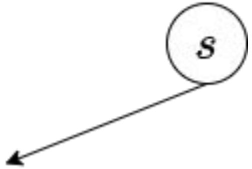
$$\text{Smoking} \in \{\text{yes}, \text{sometimes}, \text{no}\}$$

$$P(\text{Smoking} = \text{yes}) = 0.1$$

$$P(\text{Smoking} = \text{sometimes}) = 0.3$$

$$P(\text{Smoking} = \text{no}) = 0.6$$

Finding Most Likely Interpretations First



Which value to choose first ?
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$$\text{Smoking} \in \{\text{yes}, \text{sometimes}, \text{no}\}$$

$$P(\text{Smoking} = \text{yes}) = 0.1$$

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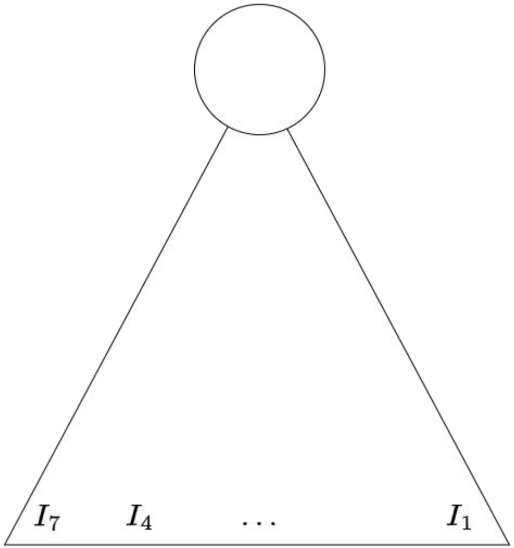
$$P(\text{Smoking} = \text{no}) = 0.6$$

Limited Discrepancy Search: Assumptions

Hypothesis 1: The value selection heuristic can be trusted (i.e., favours most likely interpretation first)

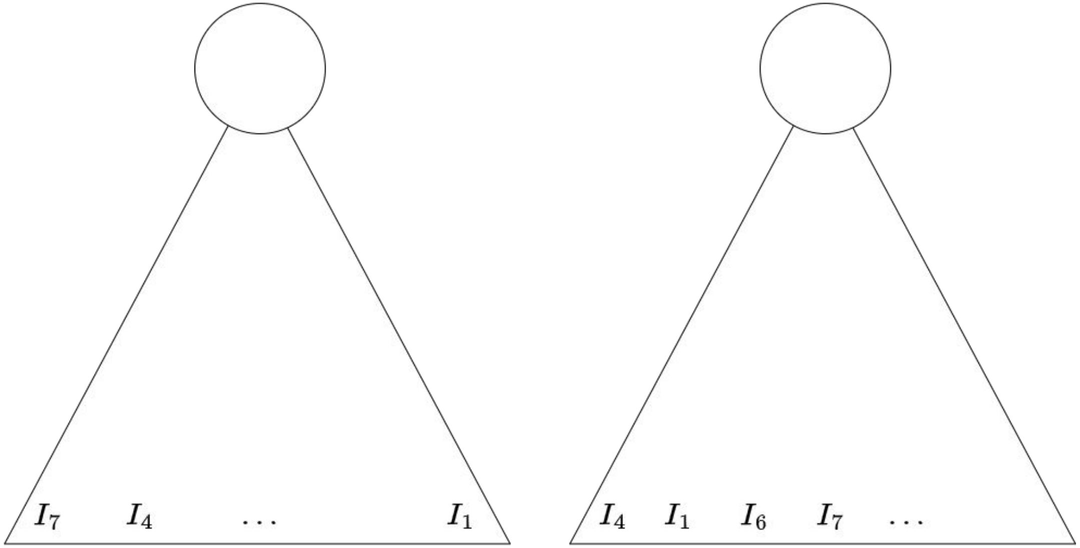
$w_{I_7} = 0.4$
$w_{I_4} = 0.2$
$w_{I_3} = 0.15$
$w_{I_6} = 0.1165$
$w_{I_5} = 0.07$
$w_{I_2} = 0.06$
$w_{I_1} = 0.0035$



Limited Discrepancy Search: Assumptions

Hypothesis 2: If the heuristics is wrong, it is only at a few nodes of the search tree

$w_{I_7} = 0.4$
$w_{I_4} = 0.2$
$w_{I_3} = 0.15$
$w_{I_6} = 0.1165$
$w_{I_5} = 0.07$
$w_{I_2} = 0.06$
$w_{I_1} = 0.0035$

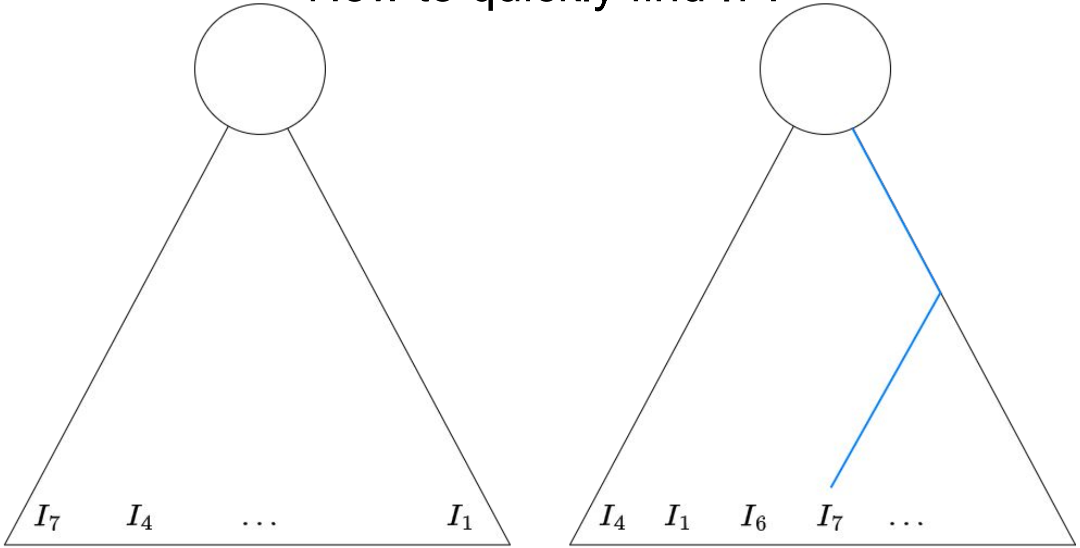


Limited Discrepancy Search: assumptions

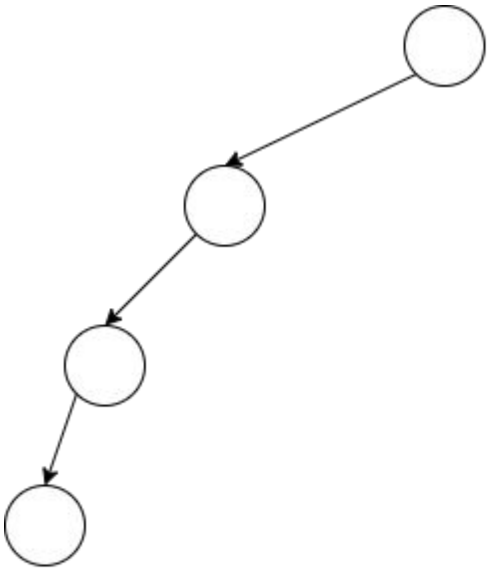
Hypothesis 2: If the heuristics is wrong, it is only at a few nodes of the search tree

$w_{I_7} = 0.4$
$w_{I_4} = 0.2$
$w_{I_3} = 0.15$
$w_{I_6} = 0.1165$
$w_{I_5} = 0.07$
$w_{I_2} = 0.06$
$w_{I_1} = 0.0035$

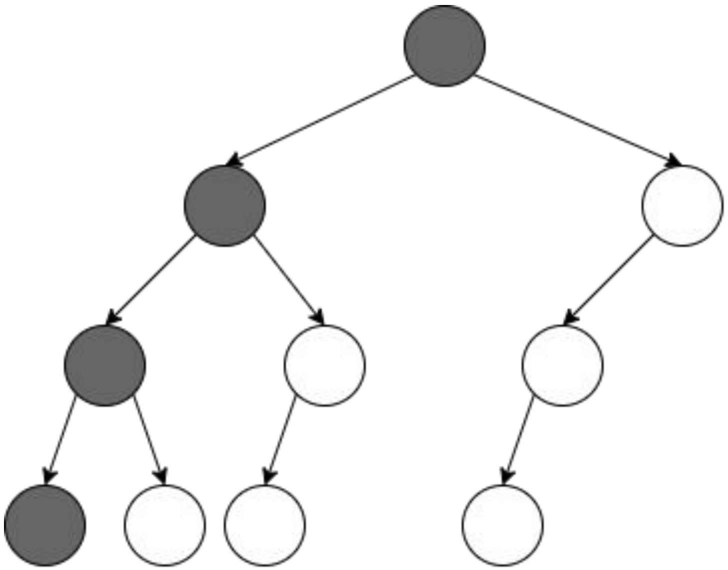
How to quickly find I7?



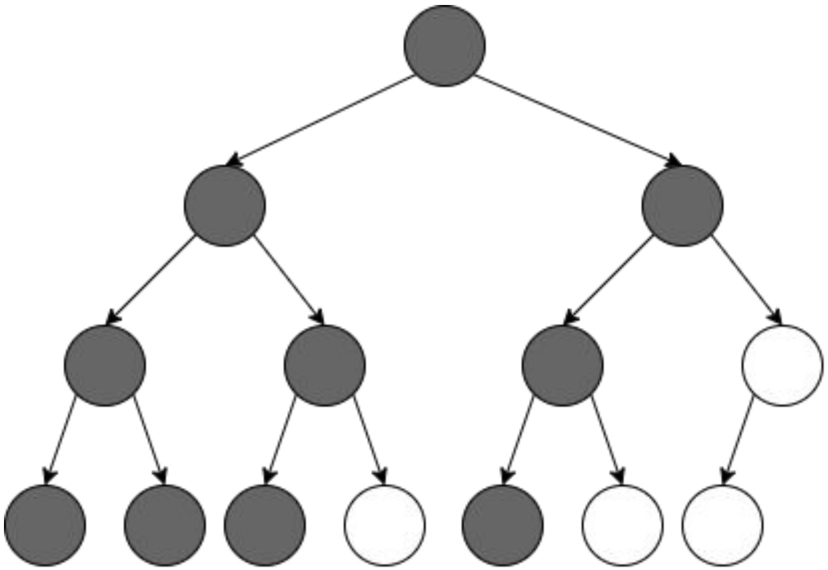
Limited Discrepancy Search: incremental search



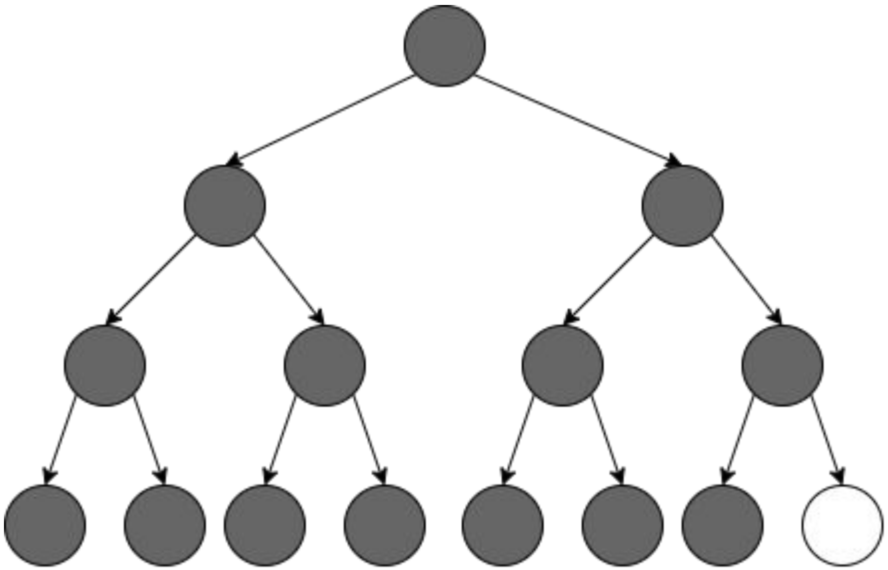
Limited Discrepancy Search: incremental search



Limited Discrepancy Search: incremental search



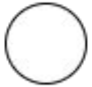
Limited Discrepancy Search: incremental search



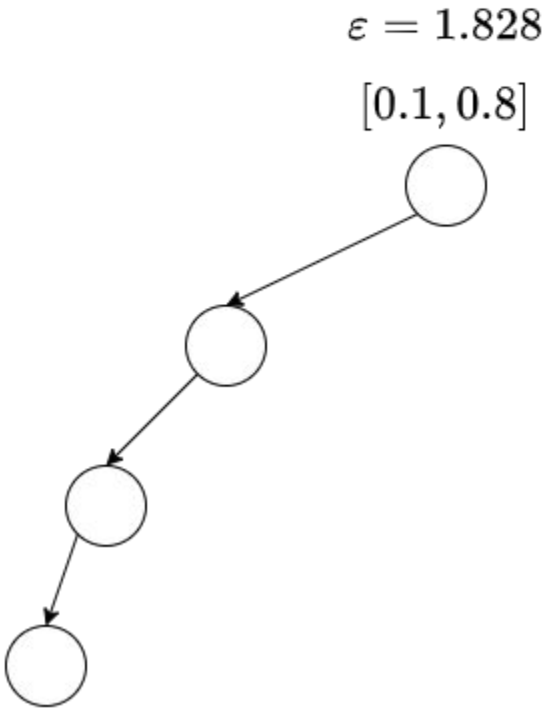
Mix it all together

$$\varepsilon = \infty$$

$$[0.0, 1.0]$$



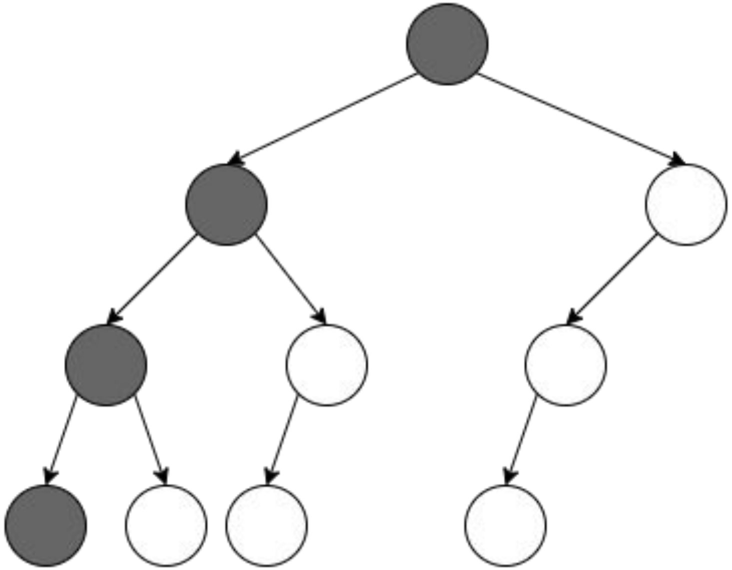
Mix it all together



Mix it all together

$$\varepsilon = 0.632$$

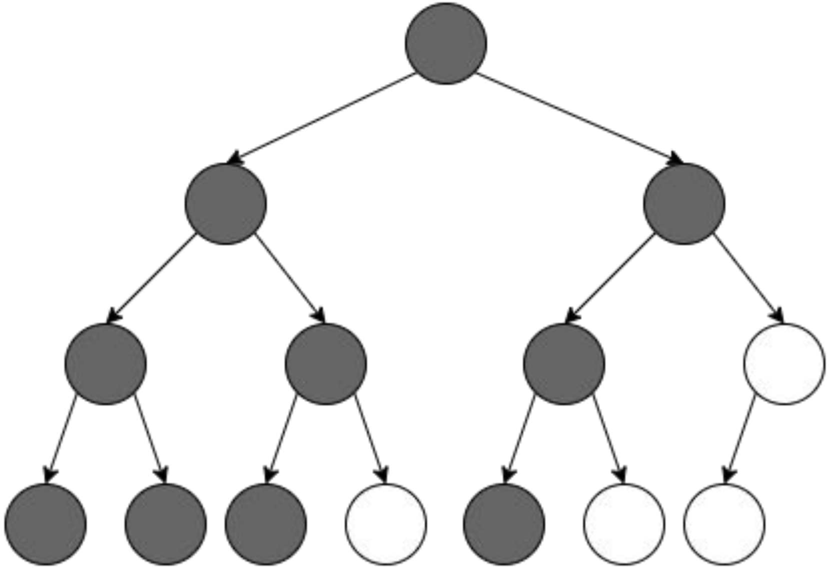
$$[0.15, 0.4]$$



Mix it all together

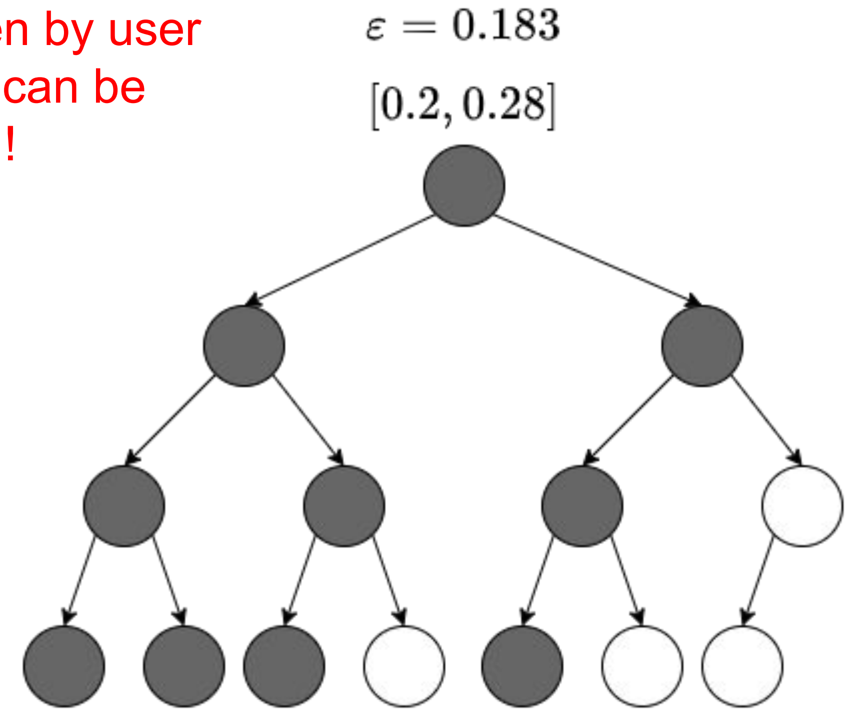
$$\varepsilon = 0.183$$

$$[0.2, 0.28]$$



Mix it all together

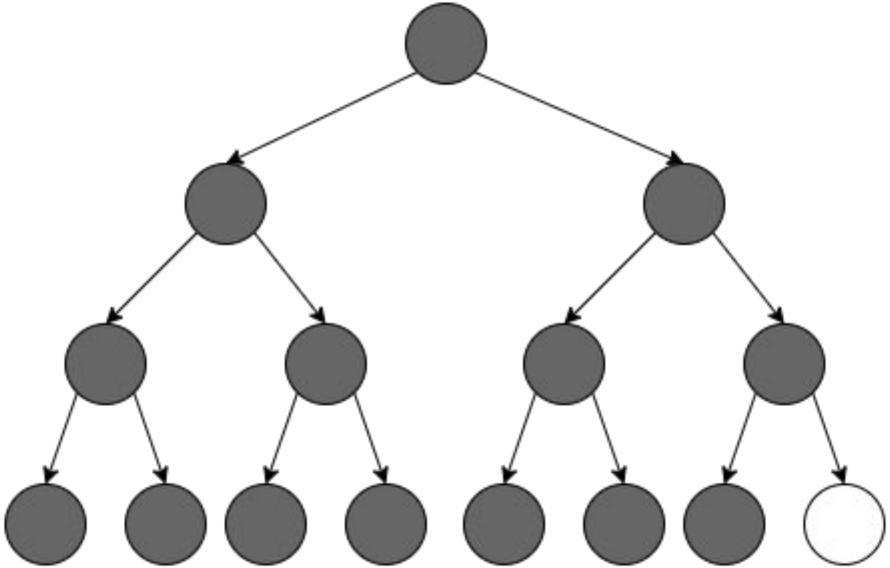
If epsilon given by user = 0.2, search can be stopped here !



Mix it all together

$$\varepsilon = 0.0$$

$$[0.2635, 0.2635]$$



Fifty shades of Approximate Inference

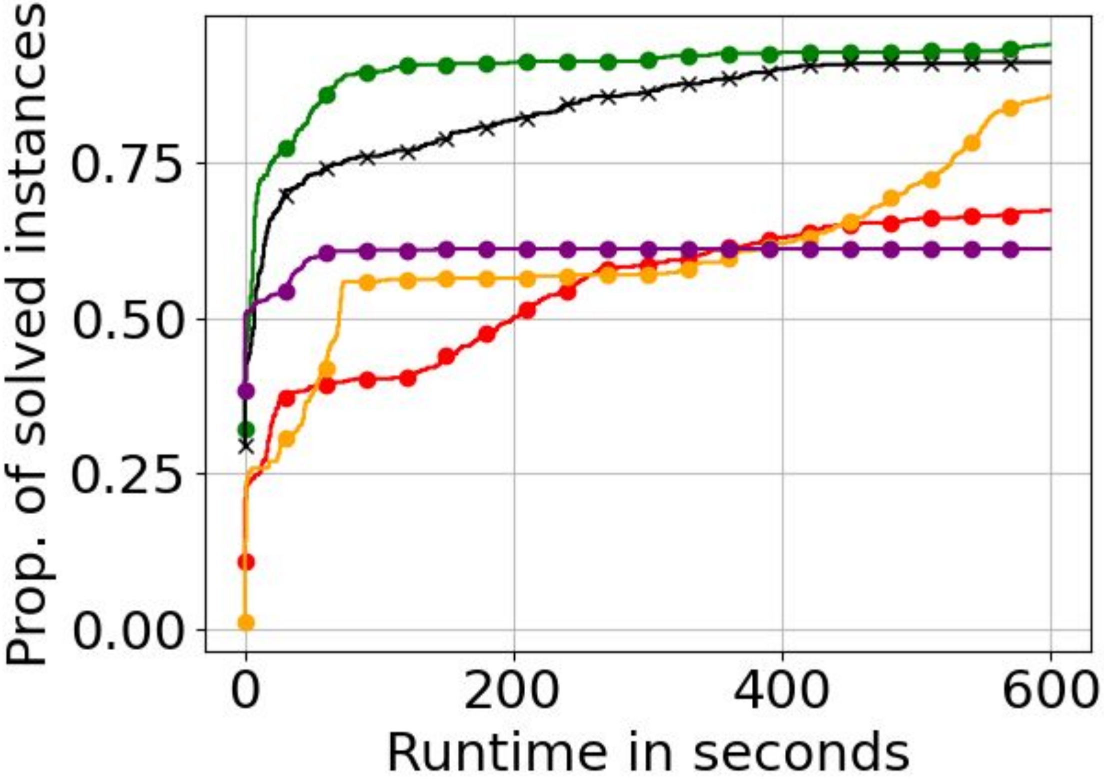
	Anytime	Guarantees	Lower bound	Upper bound
d4/gpmc/ exactMC/…	✗	Exact on completion	✗	✗
ApproxMC/ WeightMC	✗	(ε, δ)	✓	✓
PartialKC / SampleSAT	✓	Probability on the LB	✓	✓
Toulbar	✓	$(\varepsilon, 0)$	✓	✓
Schlandals-LDS	✓	$(\varepsilon, 0)$	✓	✓

Experiments

- **Marginal Probability in Bayesian Networks**, Reliability estimation in prob. graphs
- Small, large and medium networks
- Timeout: 600s
- Difficult query if solving time > 100s

- Q1. Is Schlandals-LDS efficient ?
- Q2. Better handling of difficult queries ?
- Q3. Does LDS allows faster bound convergence ?

Experiments: Cactus Plot



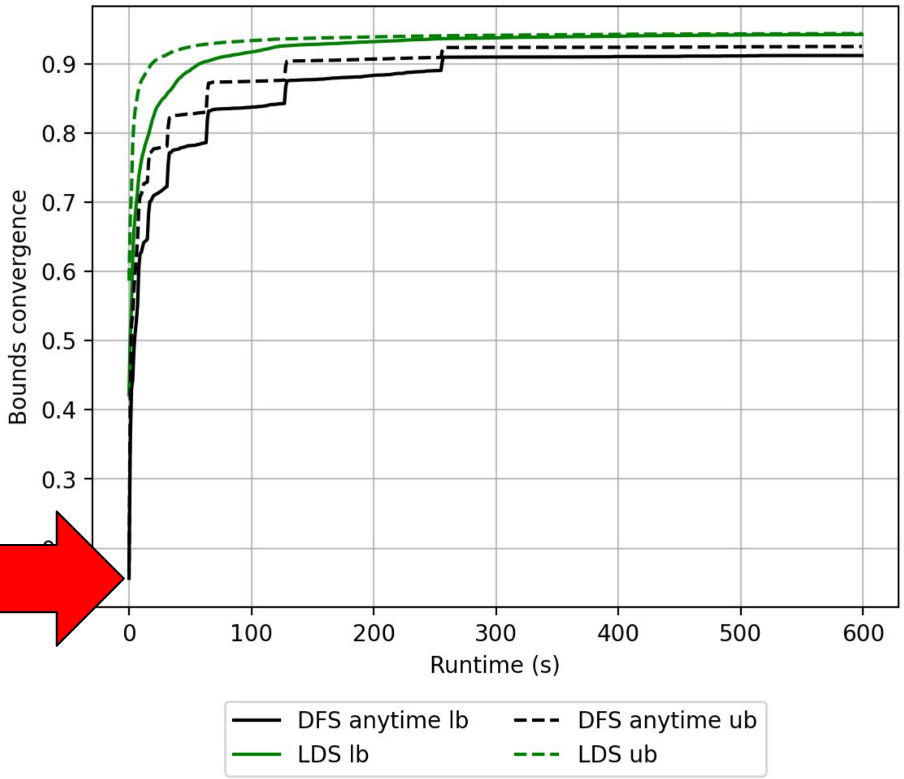
Schlandals, Schlandals-LDS(0.0)

ExactMC

d4, Toulbar

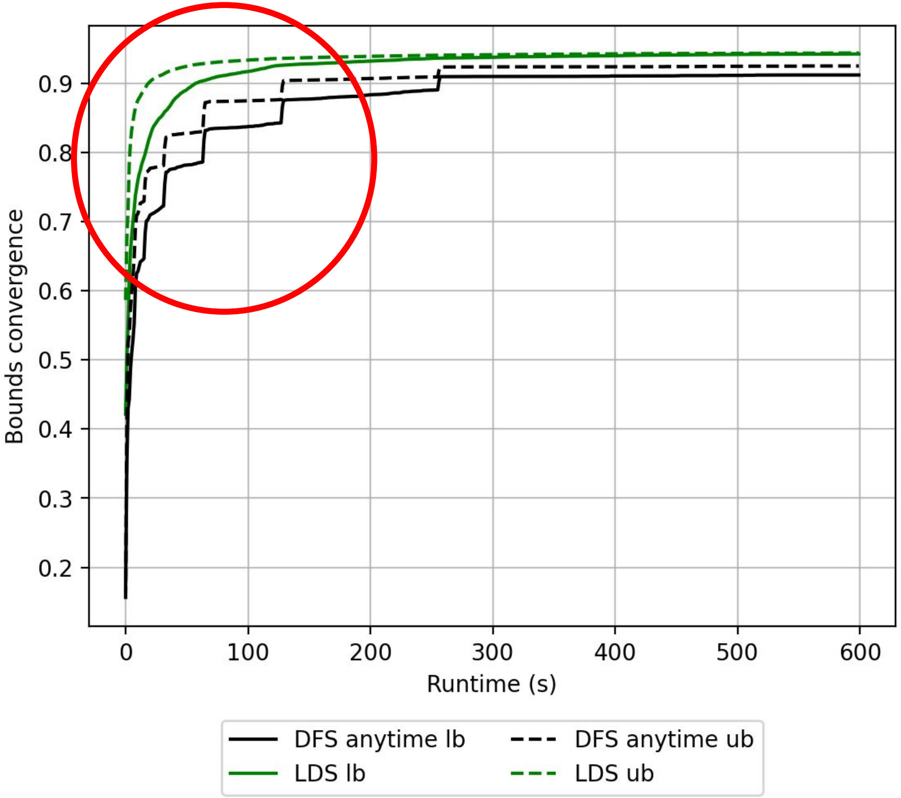
Experiments: Bounds Convergence

Far aways from the true probability

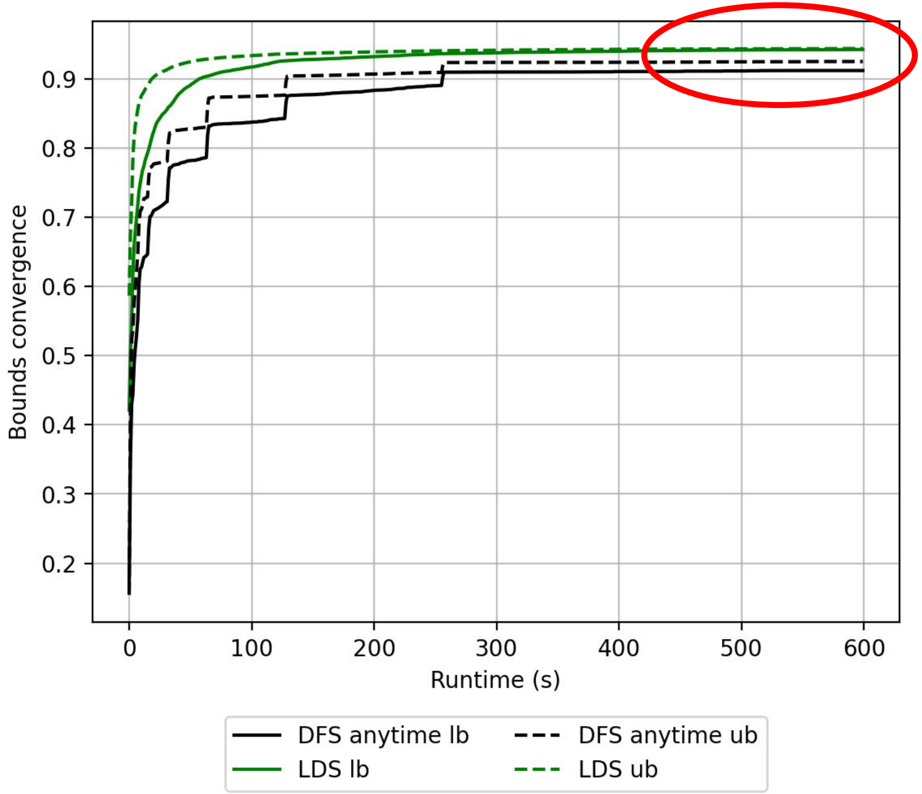


Close (< 10% gap) from the true probability

Experiments: Bounds Convergence



Experiments: Bounds Convergence



Conclusion & further work

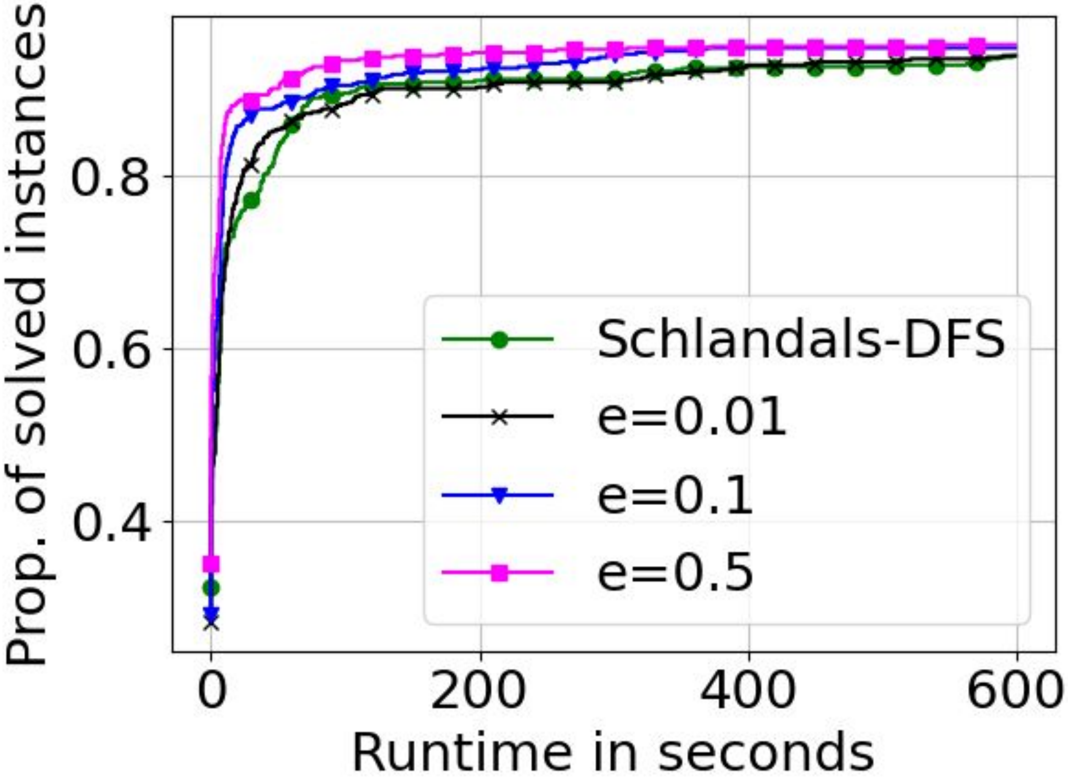
- Deterministic upper- and lower- bounds on the weighted model count
- New Anytime (projected) weighted model counter
- LDS to fasten bounds convergence

Conclusion & further work

- Deterministic upper- and lower- bounds on the weighted model count
- New Anytime (projected) weighted model counter
- LDS to fasten bounds convergence

- Change the discrepancy increment
- Use the bounds to guide the search
- Apply LDS to “classical” weighted model counter

Experiments: Cactus Plot

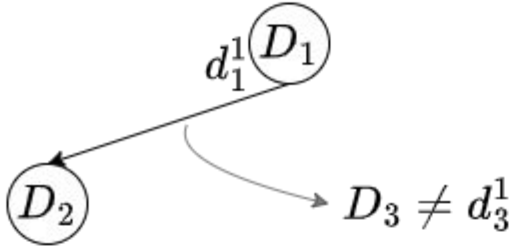


Deterministic Bounds: Lower Bound

$$WMC(F) = \sum_{j=1}^{m} P(D_i = d_i^j) \times WMC(F|_{D_i=d_i^j})$$

$$LB = \sum_{j=1}^{m'} P(D_i = d_i^j) \times WMC(F|_{D_i=d_i^j}) \; (m' <= m)$$

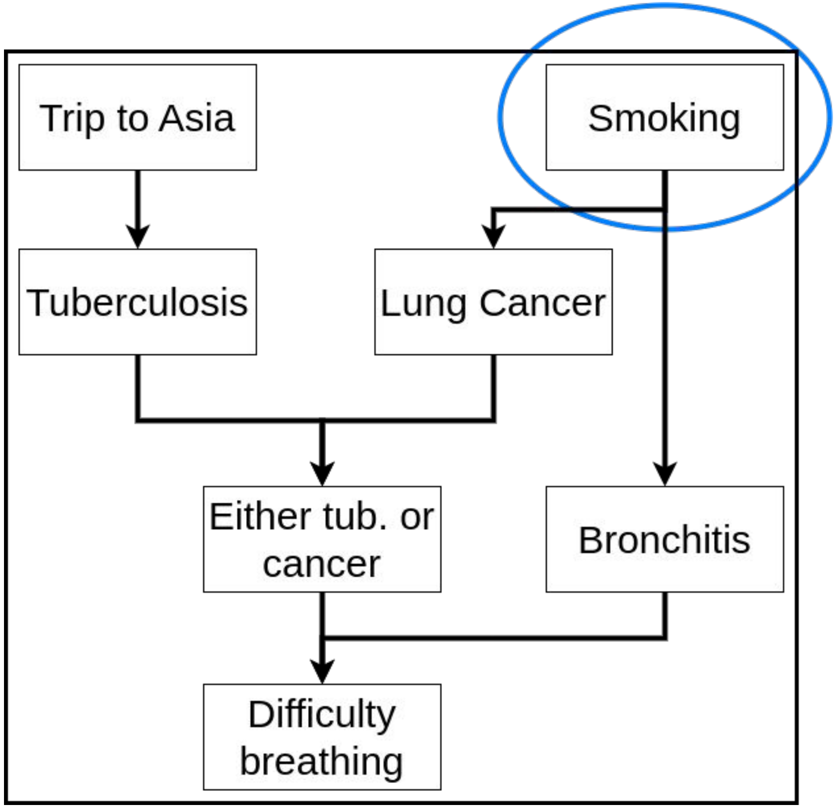
Deterministic Bounds: Upper Bound



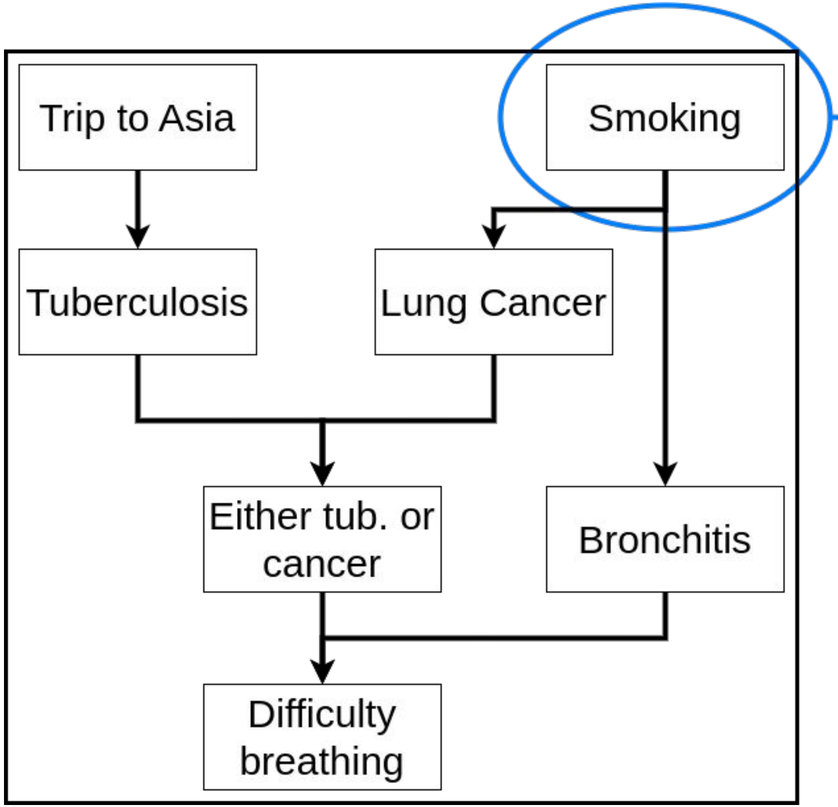
Max. before propagation - Max. after propagation

$$\prod_{D_i \in F} \left(\sum_{d \in dom(D_i)} P(D_i = d) \right) - \prod_{D_i \in F} \left(\sum_{d \in dom'(D_i)} P(D_i = d) \right)$$

Modelling Distributions in CNF

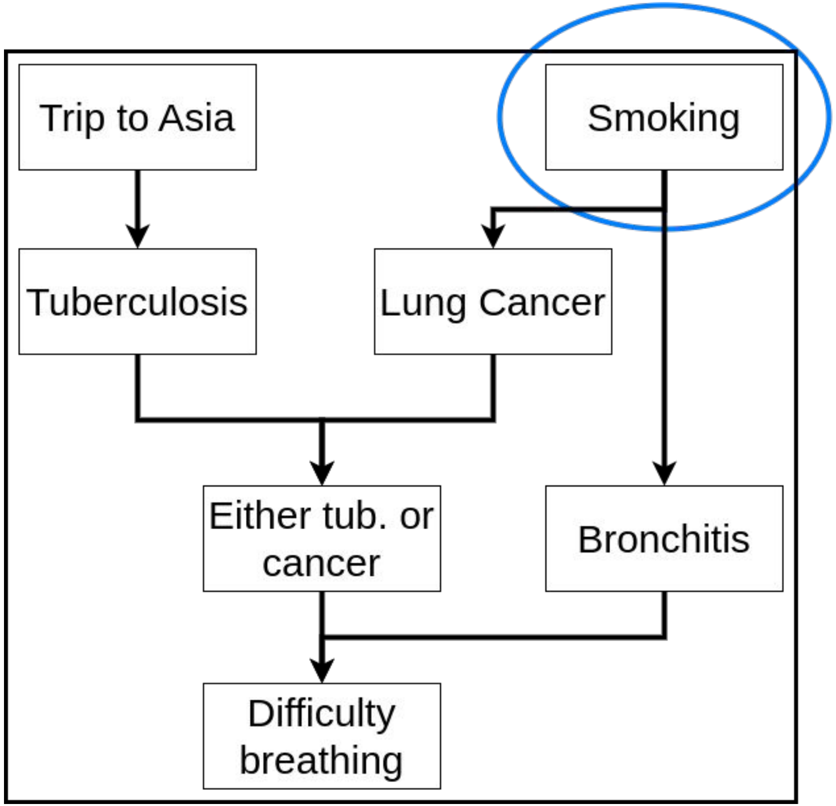


Modelling Distributions in CNF



$$\text{Smoking} \in \{\text{yes}, \text{sometimes}, \text{no}\}$$
$$P(\text{Smoking} = \text{yes}) = 0.1$$
$$P(\text{Smoking} = \text{sometimes}) = 0.3$$
$$P(\text{Smoking} = \text{no}) = 0.6$$

Modelling Distributions in CNF



$$\text{Smoking} \in \{\text{yes}, \text{sometimes}, \text{no}\}$$
$$P(\text{Smoking} = \text{yes}) = 0.1$$
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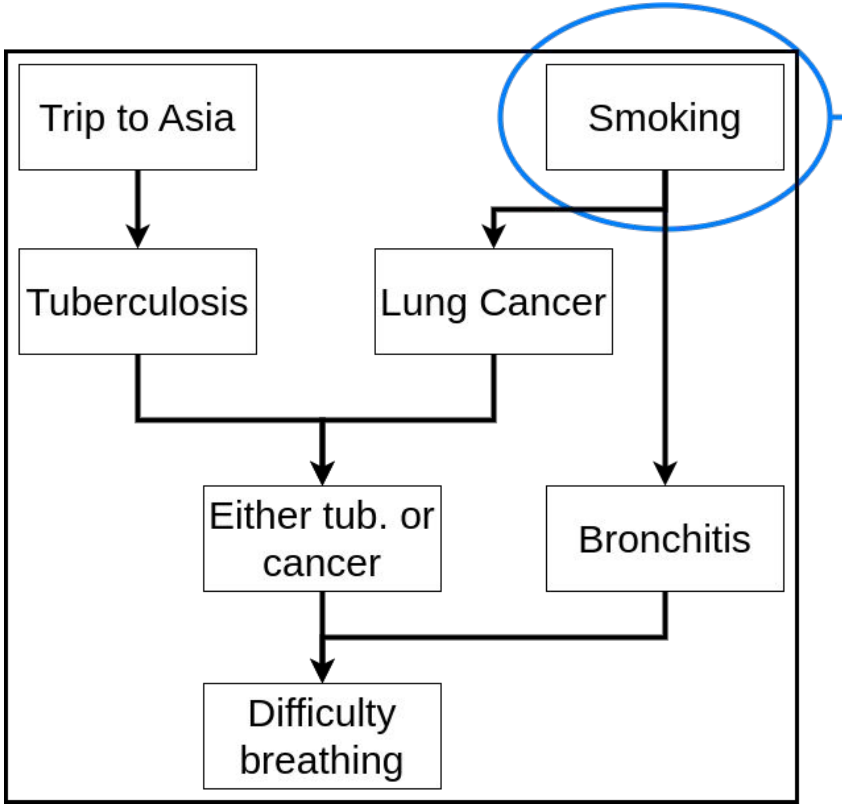
$$P(\text{Smoking} = \text{no}) = 0.6$$

$$s_{yes}, s_{sometimes}, s_{no} \in \{\top, \bot\}$$
$$\omega(s_{yes}) = 0.1, \omega(s_{sometimes}) = 0.3, \omega(s_{no}) = 0.6$$

Modelling Distributions in CNF



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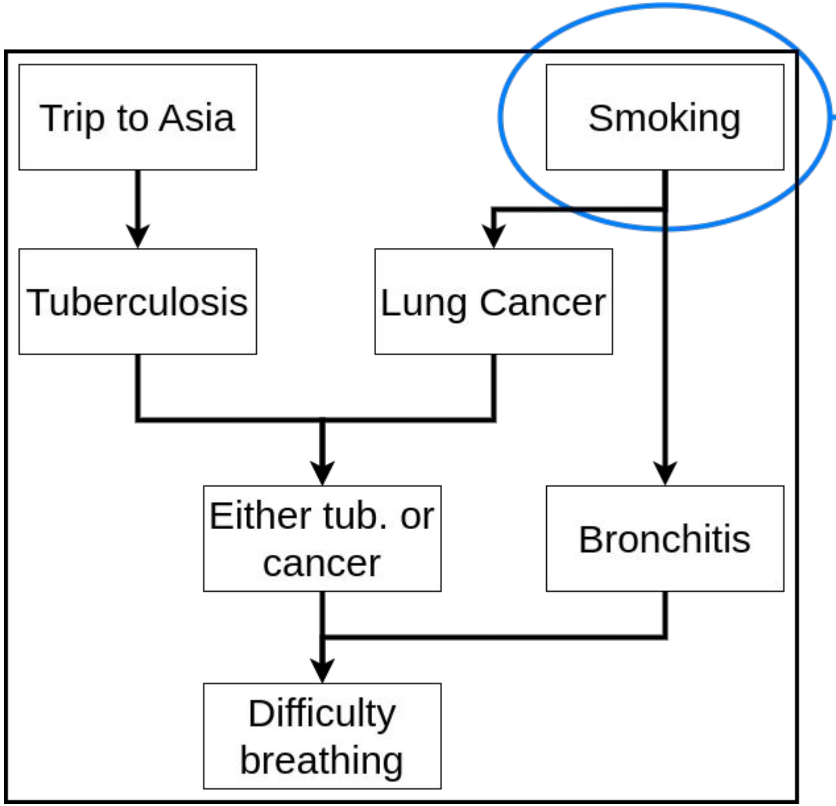
$$s_{yes}, s_{sometimes}, s_{no} \in \{\top, \bot\}$$
$$\omega(s_{yes}) = 0.1, \omega(s_{sometimes}) = 0.3, \omega(s_{no}) = 0.6$$
$$s_{yes} \vee s_{sometimes} \vee s_{no}$$
$$\neg s_{yes} \vee \neg s_{sometimes}$$
$$\neg s_{yes} \vee \neg s_{no}$$
$$\neg s_{sometimes} \vee \neg s_{no}$$

Modelling Distributions in CNF



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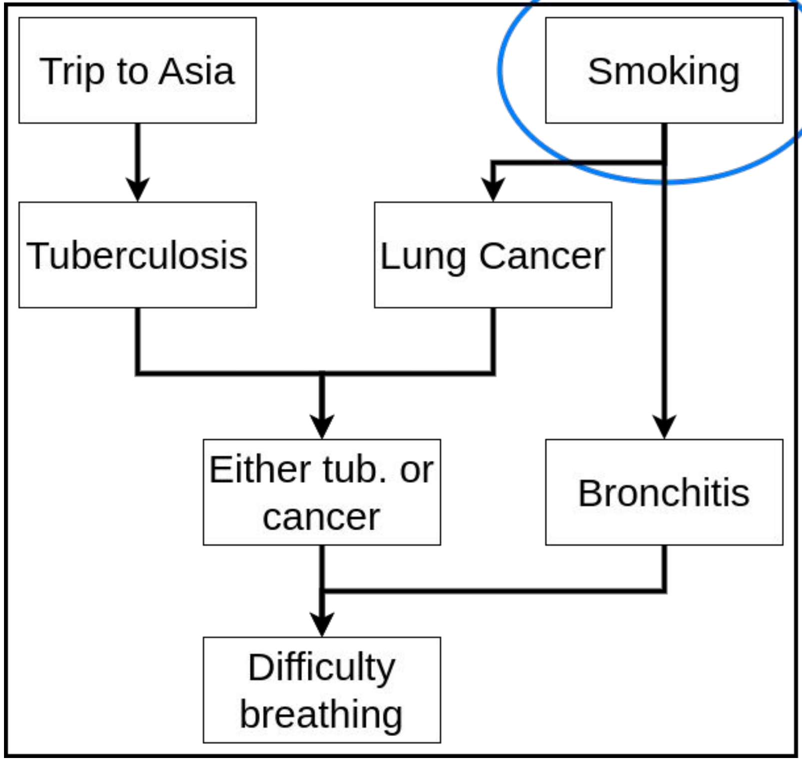
$$s_{yes}, s_{sometimes}, s_{no} \in \{\top, \bot\}$$
$$\omega(s_{yes}) = 0.1, \omega(s_{sometimes}) = 0.3, \omega(s_{no}) = 0.6$$
$$s_{yes} \vee s_{sometimes} \vee s_{no}$$ At least one
$$\neg s_{yes} \vee \neg s_{sometimes}$$
$$\neg s_{yes} \vee \neg s_{no}$$
$$\neg s_{sometimes} \vee \neg s_{no}$$

Modelling Distributions in CNF



$$\text{Smoking} \in \{\text{yes}, \text{sometimes}, \text{no}\}$$

$$P(\text{Smoking} = \text{yes}) = 0.1$$

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$$P(\text{Smoking} = \text{no}) = 0.6$$

$$s_{yes}, s_{sometimes}, s_{no} \in \{\top, \bot\}$$

$$\omega(s_{yes}) = 0.1, \omega(s_{sometimes}) = 0.3, \omega(s_{no}) = 0.6$$

$$s_{yes} \vee s_{sometimes} \vee s_{no}$$

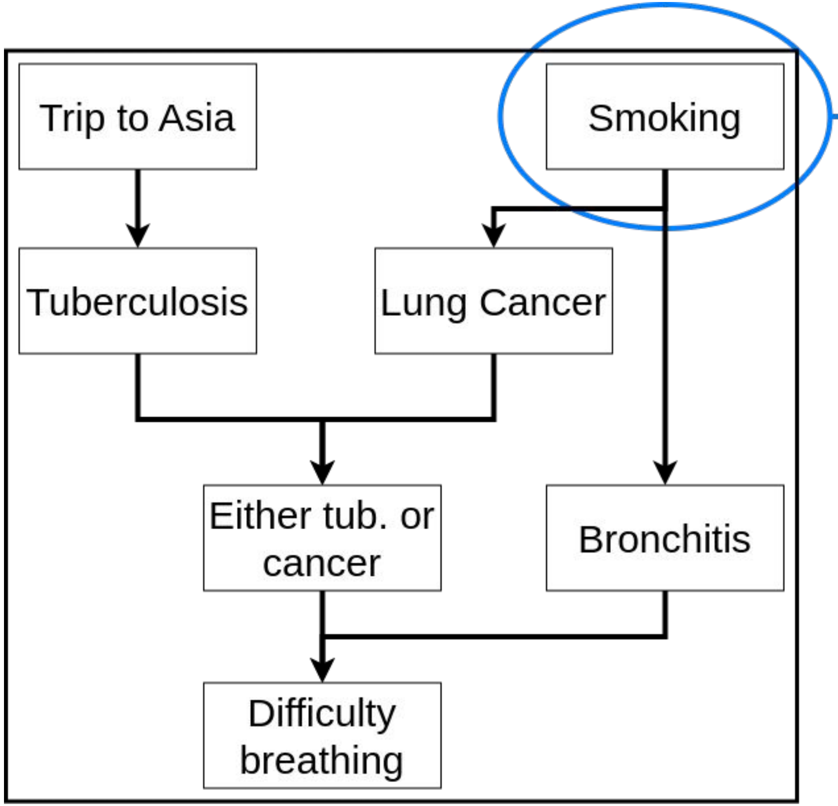
$$\neg s_{yes} \vee \neg s_{sometimes}$$

$$\neg s_{yes} \vee \neg s_{no}$$

$$\neg s_{sometimes} \vee \neg s_{no}$$

At most one

Modelling Distributions in Schlandals



$$\text{Smoking} \in \{\text{yes}, \text{sometimes}, \text{no}\}$$
$$P(\text{Smoking} = \text{yes}) = 0.1$$
$$P(\text{Smoking} = \text{sometimes}) = 0.3$$
$$P(\text{Smoking} = \text{no}) = 0.6$$

$$s_{yes}, s_{sometimes}, s_{no} \in \{\top, \bot\}$$
$$\omega(s_{yes}) = 0.1, \omega(s_{sometimes}) = 0.3, \omega(s_{no}) = 0.6$$

~~$s_{yes} \vee s_{sometimes} \vee s_{no}$~~
~~$\neg s_{yes} \vee \neg s_{sometimes}$~~
~~$\neg s_{yes} \vee \neg s_{no}$~~
~~$\neg s_{sometimes} \vee \neg s_{no}$~~

Distribution 1st-class citizens

Branching: classical CNF v.s. Schlandals