Certifying Without Loss of Generality Reasoning In Solution-Improving Maximum Satisfiability

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Joint work with Jeremias Berg, Bart Bogaerts, Jakob Nordström, Andy Oertel and Tobias Paxian

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ARTIFICIAL
INTELLIGENCE
RESEARCH GROUP

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- Software testing doesn't suffice to resolve this problem
- Formal verification techniques cannot deal with complexity of modern solvers [BHI+23]

CERTIFIED RESULTS WITH PROOF LOGGING

Design certifying algorithms [ABM+11, MMNS11] that

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Proof logging should be done

- with **minimal overhead**
- without changing a **solver's reasoning**

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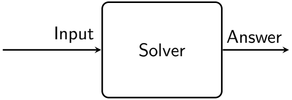
Workflow:

1. Run solver on problem input



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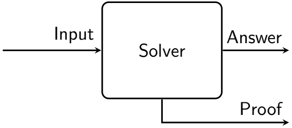

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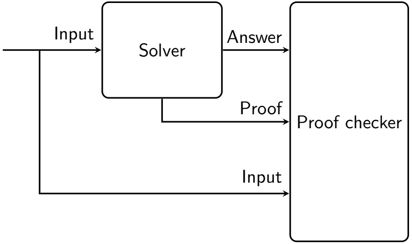

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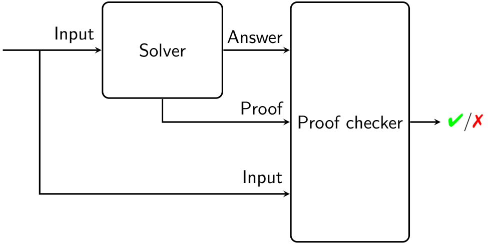

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But efficient proof logging has remained out of reach for other paradigms, e.g. Maximum Satisfiability (MaxSAT)

OUTLINE OF THIS PRESENTATION

- MaxSAT and how to certify it
- Pacose and its intricate without-loss-of-generality reasoning
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PRELIMINARIES

Example:
$$F = \{x_1 \vee x_2,\ x_2 \vee x_3,\ x_1 \vee \overline{x_2} \vee x_3\}$$

- Boolean variable: x
- Assignment α: assigns variables true $(= 1)$ or false $(= 0)$
- Literal l: variable x (satisfied if $\alpha(x) = 1$) or its negation $\overline{x}$ (satisfied if $\alpha(x) = 0$)
- Clause C: Disjunction of literals $l_1 \vee \cdots \vee l_k$
  (C is satisfied by α if at least one literal in C is assigned true)
- Propositional formula in CNF: $F = C_1 \wedge \cdots \wedge C_n$
  (F is satisfied if all clauses C_i are satisfied)

THE MAXIMUM SATISFIABILITY PROBLEM

Example:

$$F = \{x_1 \vee x_2,\ x_2 \vee x_3,\ x_1 \vee \overline{x_2} \vee x_3\}$$
$$\mathcal{O} = x_1 + x_2 + x_3$$

Optimization variant of Satisfiability Problem.

A MaxSAT-instance is a tuple $(F, \mathcal{O})$ with:

- F a propositional formula
- $\mathcal{O}$ an integer linear objective over Boolean variables

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An optimal solution is a solution such that no other solution has higher objective value.

PROOF SYSTEMS FOR MAXSAT REASONING

Proof systems for MaxSAT are studied theoretically for proof complexity

- MaxSAT resolution [LH05, HL06, BLM06, BLM07]
- Tableaux reasoning [LMS16, LCH+22, LM22]
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No certified state-of-the-art MaxSAT solver using native proof system!

MAXSAT SOLVERS

Four main categories:

- Branch-and-Bound
- Solution-Improving
- Core-Guided
- Implicit Hitting Set

Different reasoning techniques!

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Only proves answer correct, not reasoning within solver!

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- This paper – Pacose: State-Of-The-Art Solution Improving Search
  - Challenge: without-loss-of-generality reasoning in the Dynamic Polynomial Watchdog encoding

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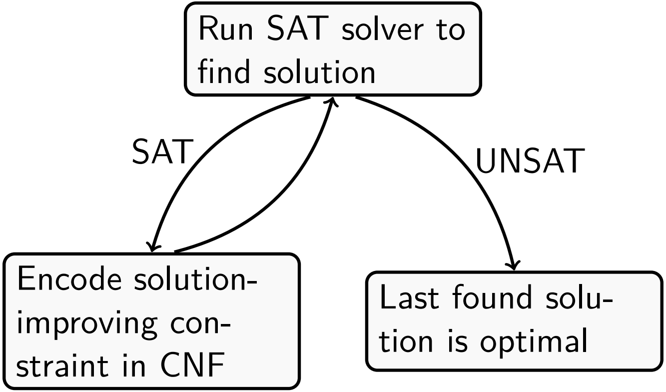
SOLUTION-IMPROVING SEARCH



HOW PACOSE WORKS

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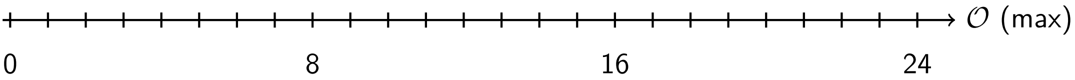

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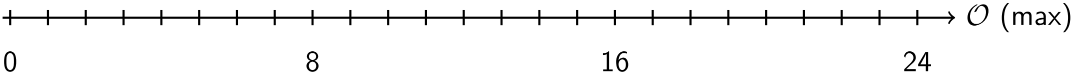

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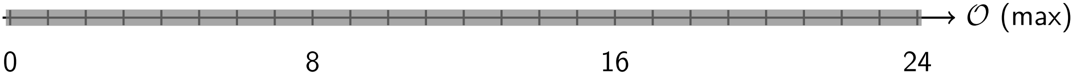

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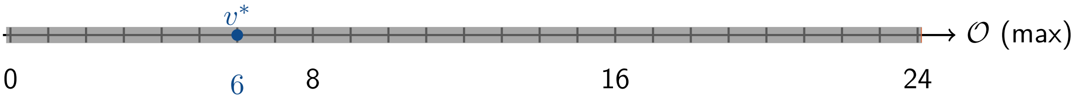

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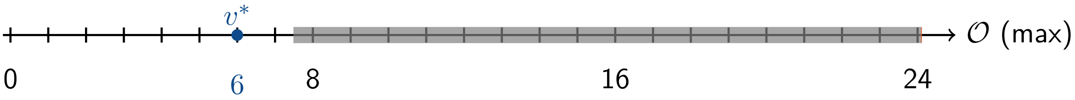

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Assume $z_1 \geq 1$

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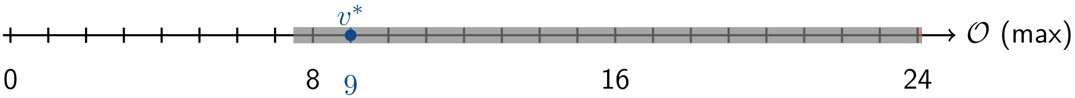

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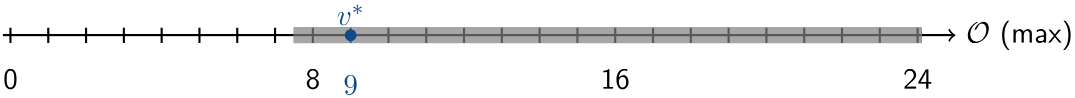

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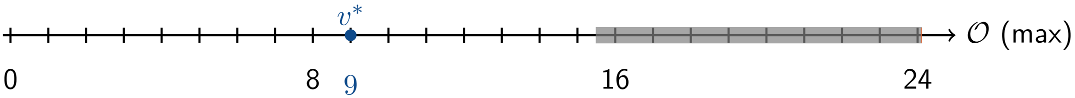

$z_2 \leftrightarrow \mathcal{O} \geq 2 \cdot 8 + T$

Assume $z_2 \geq 1$

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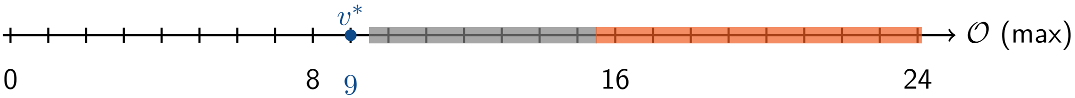

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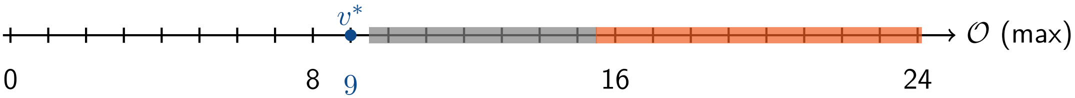

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Previously found: $z_1 \geq 1$

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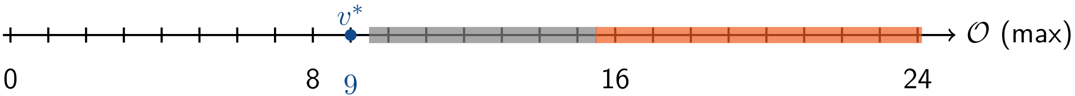

$$\mathcal{O} \geq 1 \cdot 8 + T$$

Previously found: $z_1 \geq 1$

HOW PACOSE WORKS

Introduction of variables:

- $\mathcal{Z}$ output-variables with $\text{CNF}(z_k \leftrightarrow \mathcal{O} \geq k \cdot 2^p + T)$
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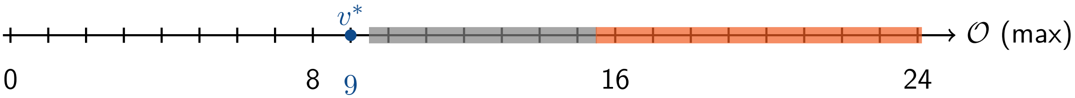

$$\mathcal{O} \geq 1 \cdot 8 + T$$

Assume $T \geq 2$

$$T = t_0 + 2t_1 + 4t_2$$

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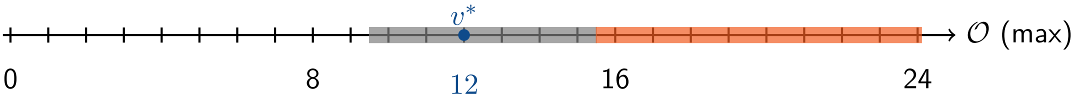

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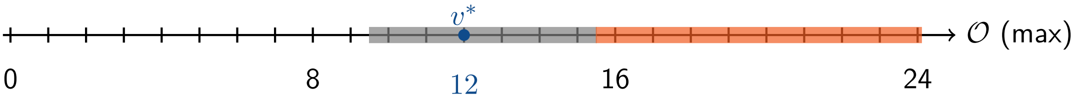

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$T = t_0 + 2t_1 + 4t_2$ $\quad$ $t_2 \geq 1$ since $T \geq 4$ (W.L.O.G. $T = 4$)

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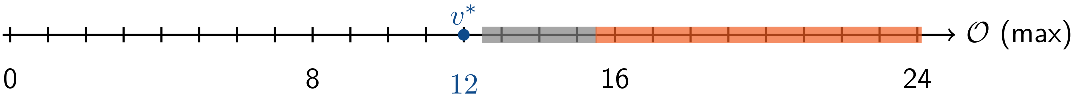

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Assume $T \geq 5$

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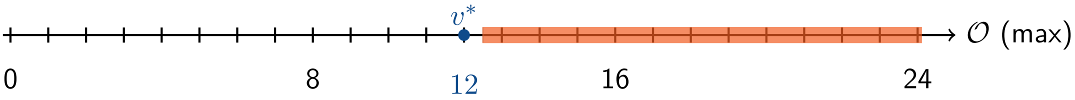

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Without loss of generality:

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Sounds about right? How to fit this in formal proof system?

The proof system VeriPB guarantees that if

- we first derive $z_1 \geq 1$ using wlog $T = 0$,
- we later derive $T \geq 4$ using wlog $T = 4$,

the second derivation will have a proof obligation that $z_1 \geq 1$ remains to hold if $T = 4$.

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VeriPB: A PROOF SYSTEM FOR PSEUDO-BOOLEAN OPTIMIZATION

VeriPB is a proof system for pseudo-Boolean optimization [BGMN22, EGMN20].

A pseudo-Boolean constraint is a 0–1 integer linear inequalities:

$$\sum_i a_i \ell_i \geq A$$

- $a_i, A \in \mathbb{Z}$
- literals ℓ_i: x_i or $\overline{x}_i$ (where $x_i + \overline{x}_i = 1$)

SOME TYPES OF PSEUDO-BOOLEAN CONSTRAINTS

1. Clauses

$$x_1 \vee \overline{x}_2 \vee x_3 \quad \Leftrightarrow \quad x_1 + \overline{x}_2 + x_3 \geq 1$$

2. Cardinality constraints

$$x_1 + x_2 + x_3 + x_4 \geq 2$$

3. General pseudo-Boolean constraints

$$x_1 + 2\overline{x}_2 + 3x_3 + 4\overline{x}_4 + 5x_5 \geq 7$$

REASONING OVER PSEUDO-BOOLEAN CONSTRAINTS USING *VeriPB*

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- Support for Optimisation [BGMN22]
  - allows deriving model-improving constraints $(\mathcal{O} > v^*)$
  - proving optimality by contradiction

WRITING DERIVATIONS TO A PROOF FILE

$$\dfrac{\dfrac{w + 2x + y \geq 2}{2w + 4x + 2y \geq 4}\text{ Multiply by 2} \qquad w + 2x + 4y + 3z \geq 5}{\dfrac{3w + 6x + 6y + 3z \geq 9}{w + 2x + 2y + 1z \geq 3}\text{ Divide by 3}}\text{ Add}$$

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Naming constraints by integers and literal axioms by the literal involved (with $\sim$ for negation) as

$$\text{Constraint 1} \doteq w + 2x + y \geq 2$$
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such a calculation is written in the proof log in reverse Polish notation as

```
pol 1 2 * 2 + 3 d
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PROOF LOGGING PACOSE

By reification: $z_1 \rightarrow \mathcal{O} \geq 8 + T$. In pseudo-Boolean, this is

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$$F \wedge \neg C \models F \upharpoonright_\omega \wedge C \upharpoonright_\omega \wedge (\mathcal{O} \upharpoonright_\omega \geq \mathcal{O})$$

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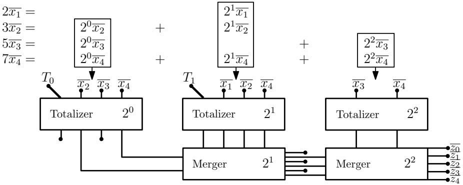
In our case:

- ω sets T to 4.

PROVING WITHOUT LOSS OF GENERALITY REASONING IN VERIPB (2)

Setting $T = 4$ breaks circuit $\mathcal{C}(\mathcal{O}, T)$ defining $\mathrm{CNF}(z_k \leftrightarrow \mathcal{O} \geq k \cdot 2^p + T)$!



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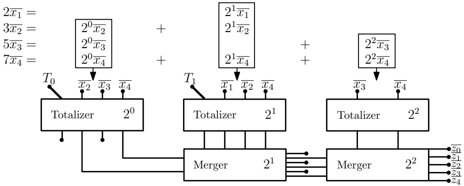

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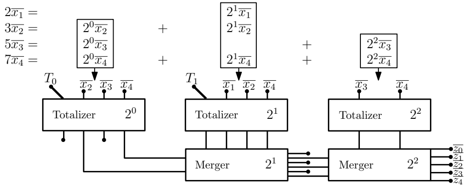

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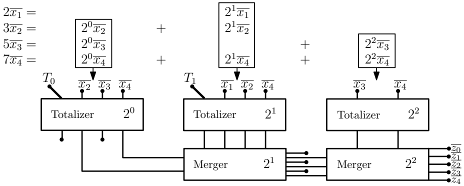

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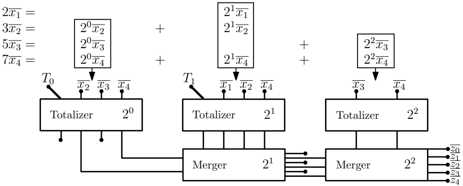

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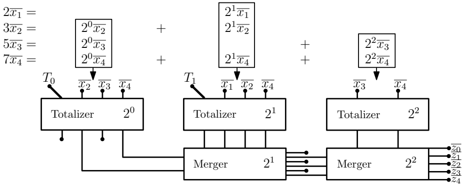
Problem: We need more expressive substitutions.
Solution: Shadow Circuits (see paper)



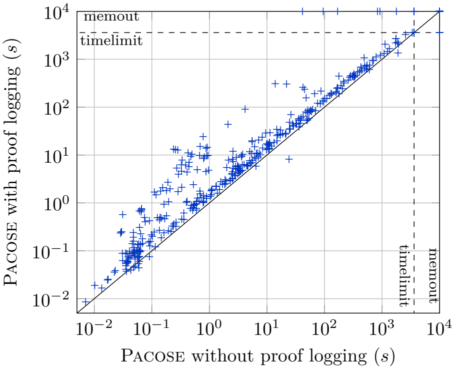
EXPERIMENTAL RESULTS

Implementation contains: TrimMaxSAT, Hardening, Binary Adder encoding

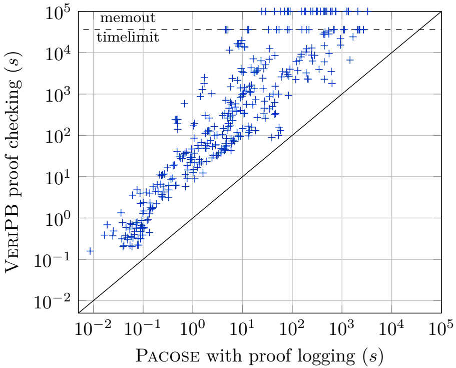
Benchmark: MaxSAT Eval. 2023 (weighted)

Resource Limits: Pacose (1h, 14GB) — VeriPB (10h, 14GB)

2/685 OoT, 9/685 OoM



29/674 OoT, 53/674 OoM



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Performance enhancement:

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Other MaxSAT Algorithms

- Branch-and-Bound solvers with clause learning [LXC+21]
- Implicit hitting sets solvers [DB11]

SUMMING UP

In this paper:

- Redundance-Based Strengthening can be used to proof log without-loss-of-generality reasoning in the Dynamic-Polynomial Watchdog ,
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Thank you for your attention!

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