

Cumulative Scheduling with Calendars and Overtime

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LAVAL

Resource Constrained Project Scheduling Problem

Schedule tasks subject to :

- Precedence constraints
- Resource usage constraints

Resource Constrained Project Scheduling Problem

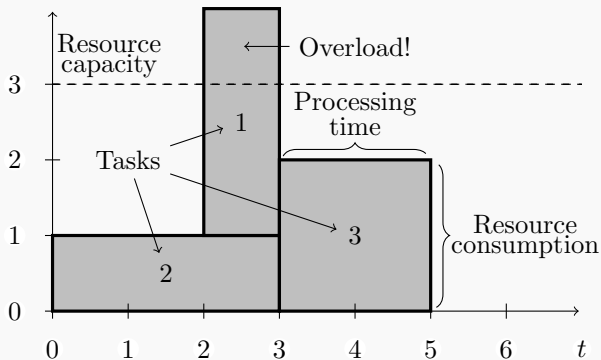
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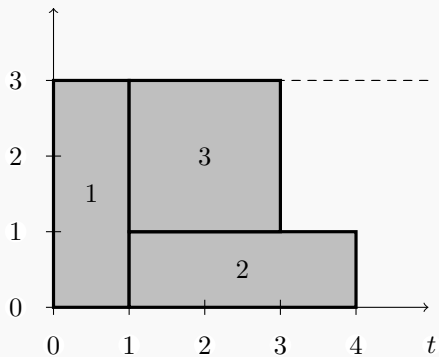
Definition of task $i \in \mathcal{I}$

- S_i : Starting time variable
- p_i : Processing time
- h_i : Resource usage

Resource usage constraints



Resource usage constraints



The CUMULATIVE constraint models this.

Calendars with Scheduled Overtime

Motivation

- In practice, not all time points are worked the same (such as weekends)
- Some time points may represent work hours that are out of the regular work schedule.

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Inspiration




A Constraint Programming Approach to Ship Refit Project Scheduling, Boudreault et. al. (CP22) :

- Overtime is available to finish tasks earlier
- Every task follows some calendar periodic on weeks

Calendars with Scheduled Overtime

In our generalization, calendars are arbitrary.




Symbol definitions

-  : Closed time point
-  : Regular time point
-  : Overtime time point

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Example of a calendar

A 8-16 work day with a 4 overtime hour time slot :



Effects of calendars with overtime

New variables

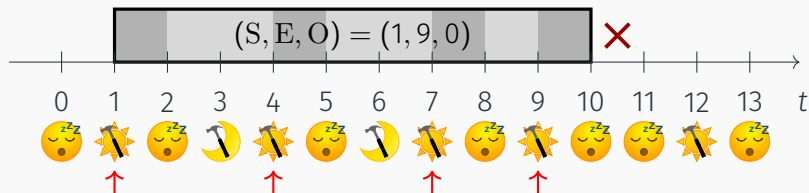
- E_j : Elapsed time (actual duration)
- O_j : Amount of overtime used

Rules

- A task may not start nor end with an unworked hour
- Time worked in an execution window is exactly the processing time of the task
- We cannot work more overtime than available in the execution window

Effects of calendars with overtime

Task of processing time 4



Domain variation

Initial domains :

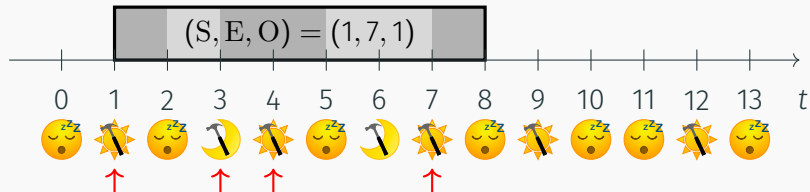
$\text{dom}(S) = \{0..8\}$, $\text{dom}(E) = \{0..8\}$, and $\text{dom}(O) = \{0..8\}$

Filtered domain :

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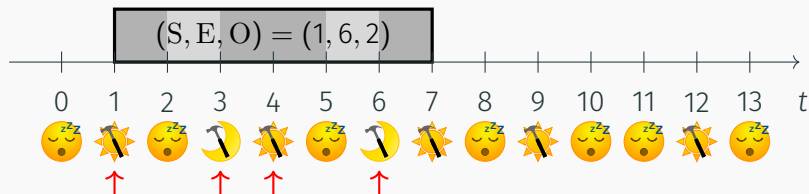
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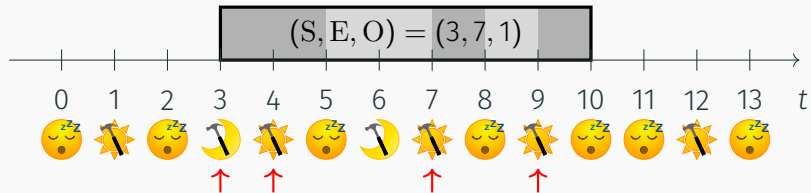
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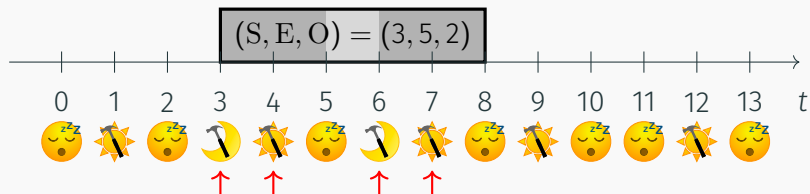
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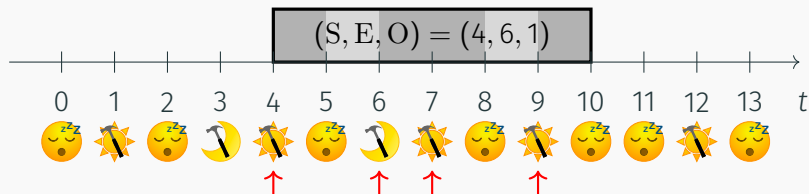
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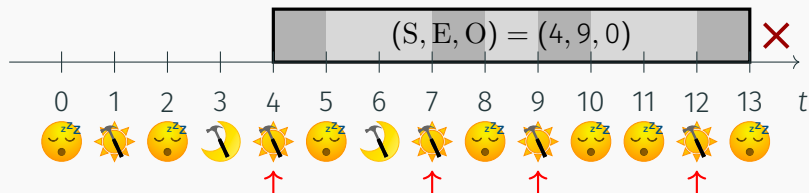
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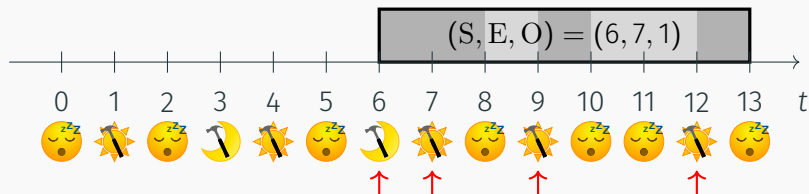
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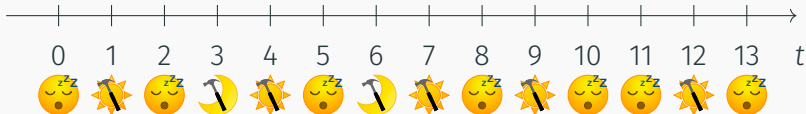
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Filtered domain :

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Effects of calendars with overtime

Task of processing time 4



Domain variation

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Filtered domain :

$\text{dom}(S) = \{1..6\}$, $\text{dom}(E) = \{5..7\}$, and $\text{dom}(O) = \{1..2\}$

First Contribution - The CALENDAROVERTIME constraint

Definition

$$\begin{aligned} \text{CALENDAROVERTIME}(S, E, O, p, \text{Cal}) &\stackrel{\text{def}}{\iff} \text{Cal}[S] \neq \text{🍌} \\ &\wedge \text{Cal}[S + E - 1] \neq \text{🍌} \\ &\wedge O = p - |\{t \in [S, S + E) \mid \text{Cal}[t] = \text{🍌}\}| \\ &\wedge O \leq |\{t \in [S, S + E) \mid \text{Cal}[t] = \text{🍌}\}| \\ &\wedge |\{t \in \{S, S + E - 1\} \mid \text{Cal}[t] = \text{🍌}\}| \leq O \end{aligned}$$

Context

- One constraint per task
- Does not deal with resource usages
- Its propagator applies bounds consistency

Key data-structures for the propagator

- We want to perform fast propagation
- We precompute four arrays that permit performing various computations in constant time

$$X \begin{bmatrix} -1 \end{bmatrix}$$



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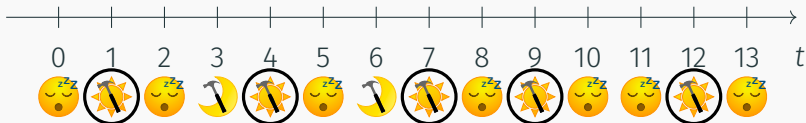
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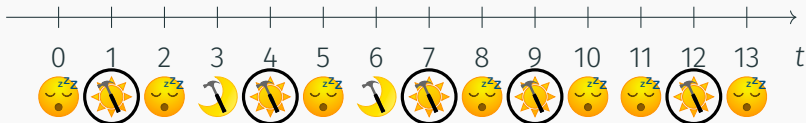
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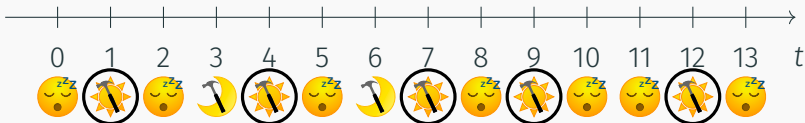


Key data-structures for the propagator

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X

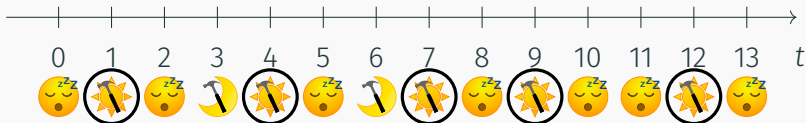
-1	0	0	0	1
----	---	---	---	---



Key data-structures for the propagator

- We want to perform fast propagation
- We precompute four arrays that permit performing various computations in constant time

X	-1	0	0	0	1	1	1	2	2	3	3	3	4	4
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---



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X	-1	0	0	0	1	1	1	2	2	3	3	3	4	4
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

Y	1
---	---



Key data-structures for the propagator

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X	-1	0	0	0	1	1	1	2	2	3	3	3	4	4
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

Y	1	4
---	---	---

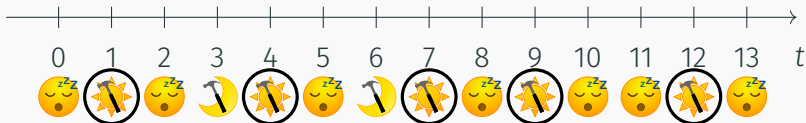


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---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

Y	1	4	7
---	---	---	---



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X	-1	0	0	0	1	1	1	2	2	3	3	3	4	4
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

Y	1	4	7	9
---	---	---	---	---

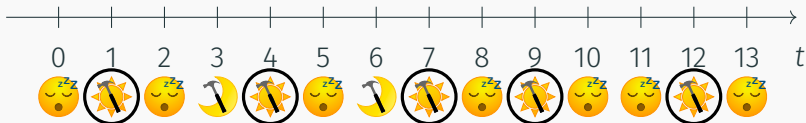


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---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

Y	1	4	7	9	12
---	---	---	---	---	----



Utility functions

Counting the regular time points in an interval

X	-1	0	0	0	1	1	1	2	2	3	3	3	4	4
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

Y	1	4	7	9	12
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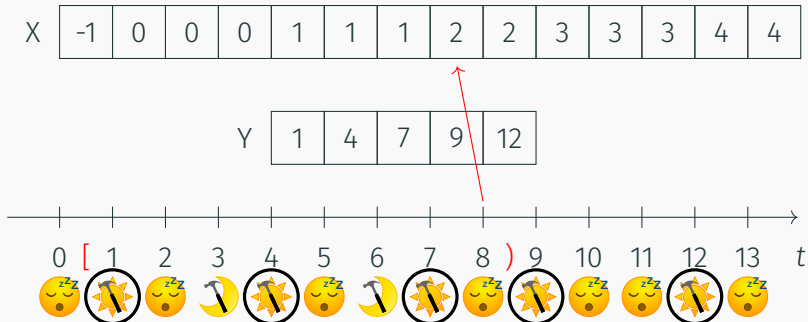
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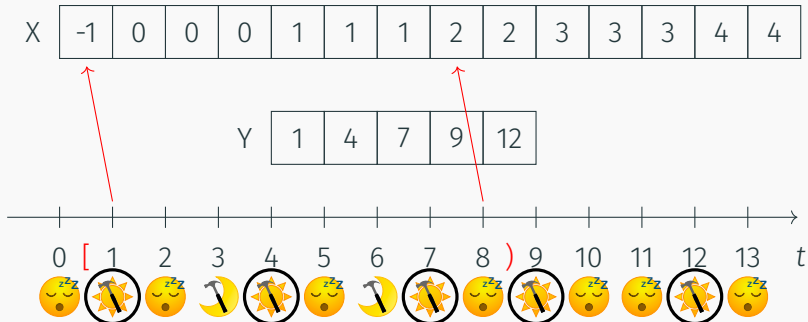
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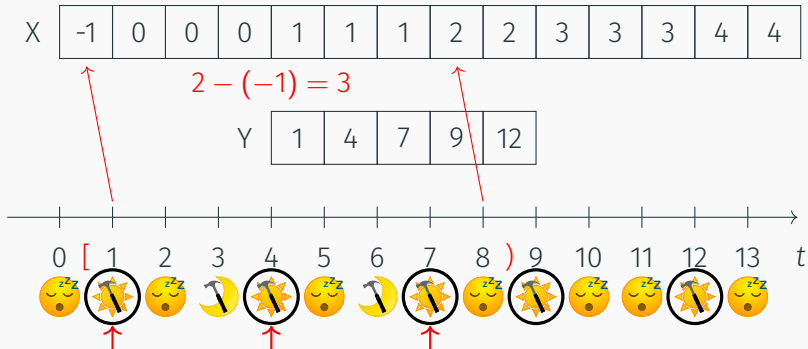
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Utility functions

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Utility functions

Finding the (non-strict) previous regular time point from any point.

X	-1	0	0	0	1	1	1	2	2	3	3	3	4	4
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

Y	1	4	7	9	12
---	---	---	---	---	----



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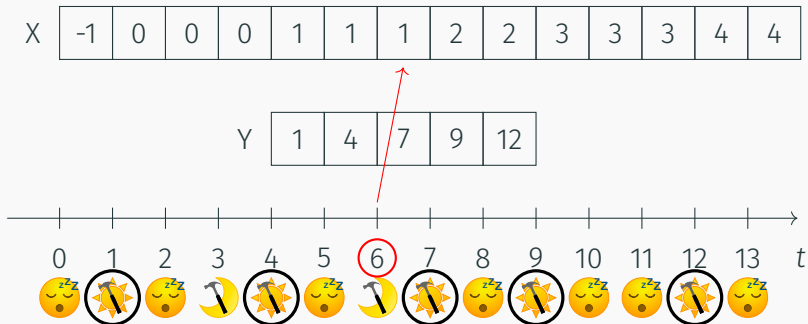
X	-1	0	0	0	1	1	1	2	2	3	3	3	4	4
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

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---	---	---	---	---	----



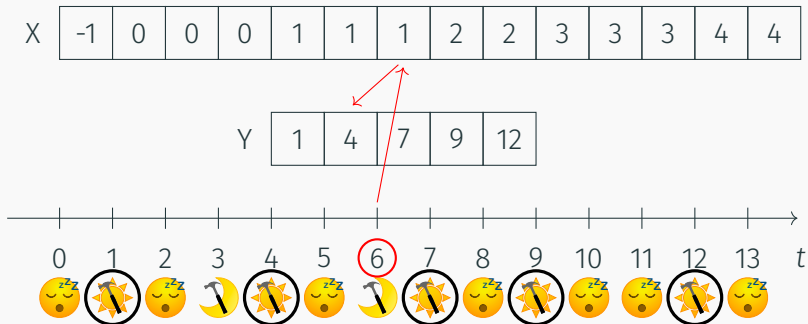
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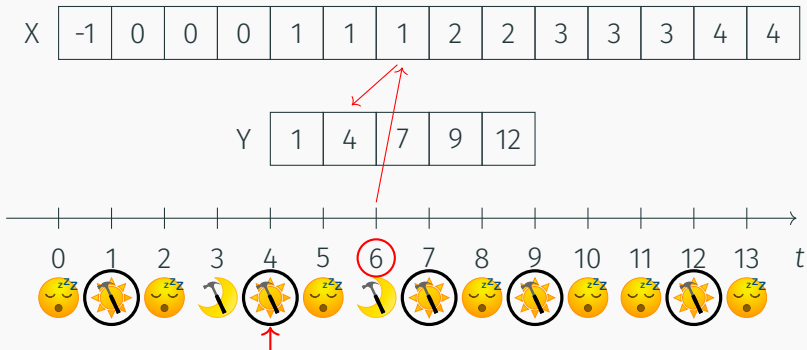
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Utility functions

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Utility functions

Finding the smallest end for an interval containing a given amount of regular time

X	-1	0	0	0	1	1	1	2	2	3	3	3	4	4
---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

Y	1	4	7	9	12
---	---	---	---	---	----



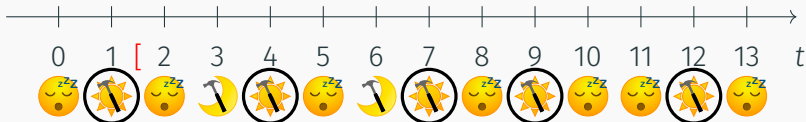
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---	----	---	---	---	---	---	---	---	---	---	---	---	---	---

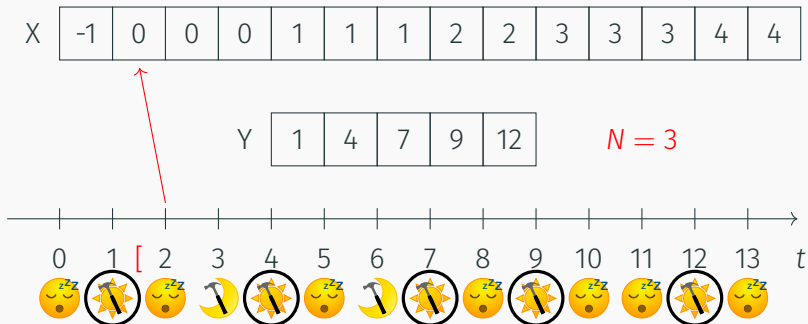
Y	1	4	7	9	12
---	---	---	---	---	----

$N = 3$



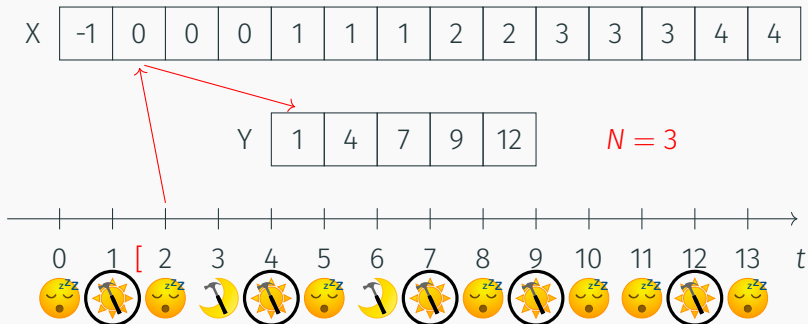
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Finding the smallest end for an interval containing a given amount of regular time



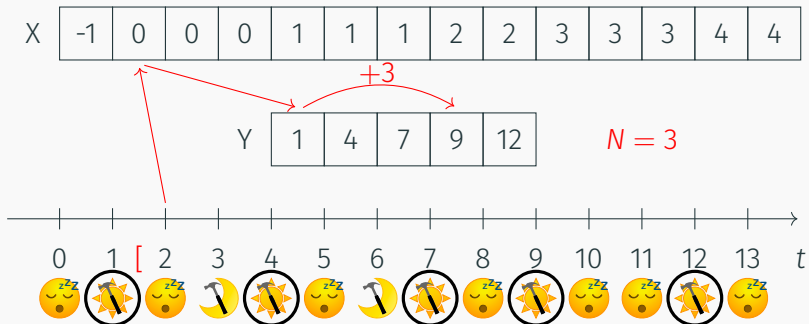
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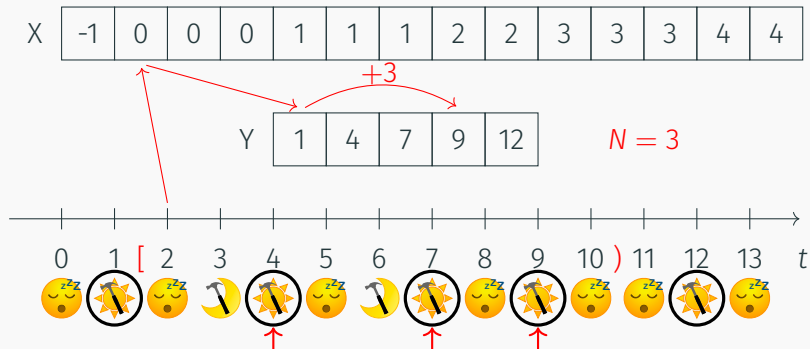
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Steps to create a minimal execution window for a given S

1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
3. Add any missing regular time (verify \overline{O})
4. Correct any potential unworked overtime tail
5. Verify \overline{E} , \underline{O} , and the unworked overtime head/tail

Propagation trace

Filtering the lower bound of S , for a task with $p = 2$

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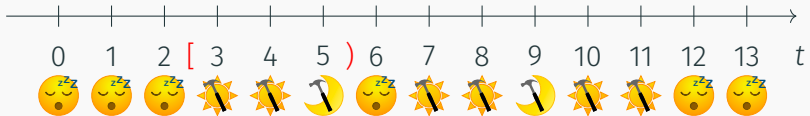
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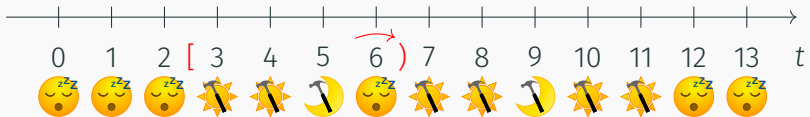
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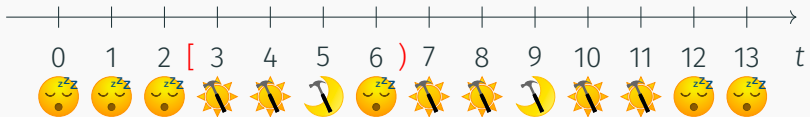
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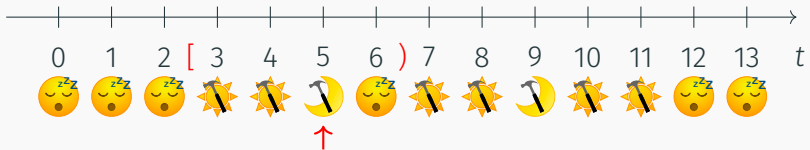
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Not Ok



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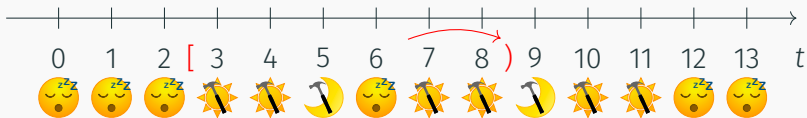
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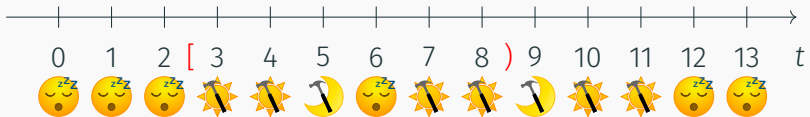
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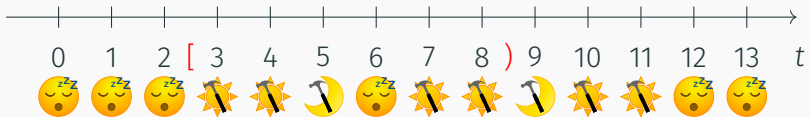
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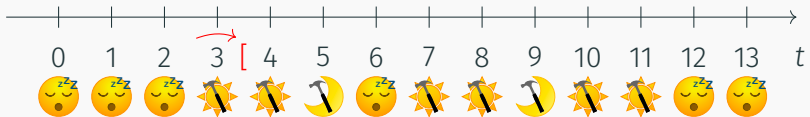
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Propagation trace

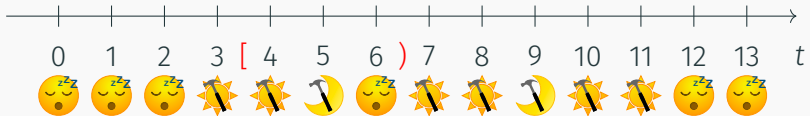
Filtering the lower bound of S , for a task with $p = 2$

1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
3. Add any missing regular time (verify \overline{O})
4. Correct any potential unworked overtime tail
5. Verify \overline{E} , \underline{O} , and the unworked overtime head/tail

$\text{dom}(S) = \{0..8\}$

$\text{dom}(E) = \{3..6\}$

$\text{dom}(O) = \{0..1\}$



Propagation trace

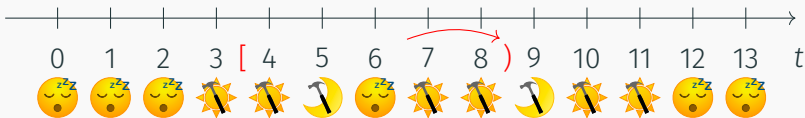
Filtering the lower bound of S , for a task with $p = 2$

1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
3. Add any missing regular time (verify \overline{O})
4. Correct any potential unworked overtime tail
5. Verify \overline{E} , \underline{O} , and the unworked overtime head/tail

$\text{dom}(S) = \{0..8\}$

$\text{dom}(E) = \{3..6\}$

$\text{dom}(O) = \{0..1\}$



Propagation trace

Filtering the lower bound of S , for a task with $p = 2$

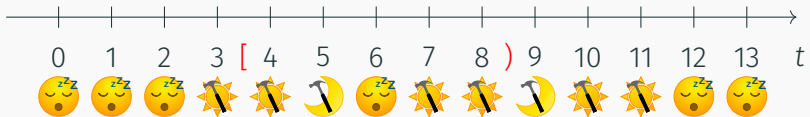
1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
3. Add any missing regular time (verify \overline{O})
4. Correct any potential unworked overtime tail
5. Verify \overline{E} , \underline{O} , and the unworked overtime head/tail

$\text{dom}(S) = \{0..8\}$

$\text{dom}(E) = \{3..6\}$

$\text{dom}(O) = \{0..1\}$

Ok



Propagation trace

Filtering the lower bound of S , for a task with $p = 2$

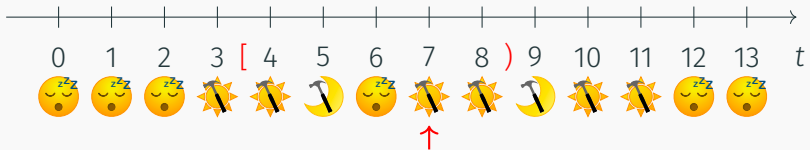
1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
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$\text{dom}(E) = \{3..6\}$

$\text{dom}(O) = \{0..1\}$

Ok



Propagation trace

Filtering the lower bound of S , for a task with $p = 2$

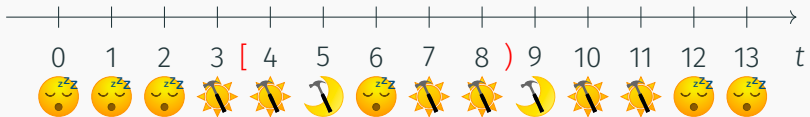
1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
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5. Verify \overline{E} , \underline{O} , and the unworked overtime head/tail

$\text{dom}(S) = \{0..8\}$

$\text{dom}(E) = \{3..6\}$

$\text{dom}(O) = \{0..1\}$

Ok



Propagation trace

Filtering the lower bound of S , for a task with $p = 2$

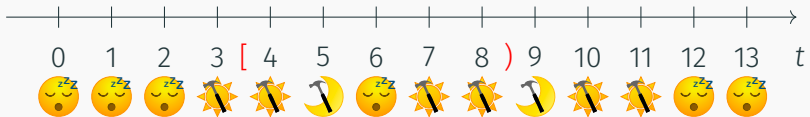
1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
3. Add any missing regular time (verify \overline{O})
4. Correct any potential unworked overtime tail
5. Verify \overline{E} , \underline{O} , and the unworked overtime head/tail

$\text{dom}(S) = \{0..8\}$

$\text{dom}(E) = \{3..6\}$

$\text{dom}(O) = \{0..1\}$

Ok



Propagation trace

Filtering the lower bound of S , for a task with $p = 2$

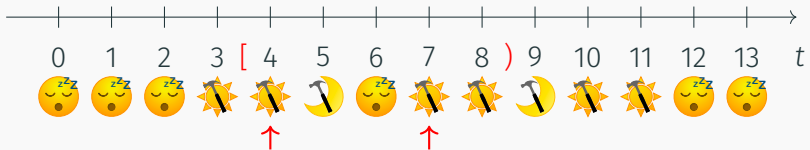
1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
3. Add any missing regular time (verify \overline{O})
4. Correct any potential unworked overtime tail
5. Verify \overline{E} , \underline{O} , and the unworked overtime head/tail

$\text{dom}(S) = \{0..8\}$

$\text{dom}(E) = \{3..6\}$

$\text{dom}(O) = \{0..1\}$

Ok



Propagation trace

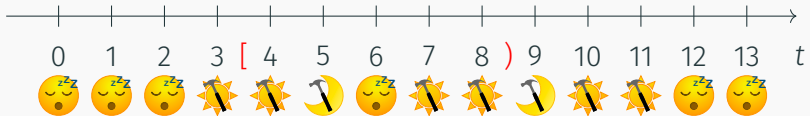
Filtering the lower bound of S , for a task with $p = 2$

1. Add p non-closed time points
2. Adjust to reach the minimal duration (verify \underline{E})
3. Add any missing regular time (verify \overline{O})
4. Correct any potential unworked overtime tail
5. Verify \overline{E} , \underline{O} , and the unworked overtime head/tail

$\text{dom}(S) = \{4..8\}$

$\text{dom}(E) = \{3..6\}$

$\text{dom}(O) = \{0..1\}$

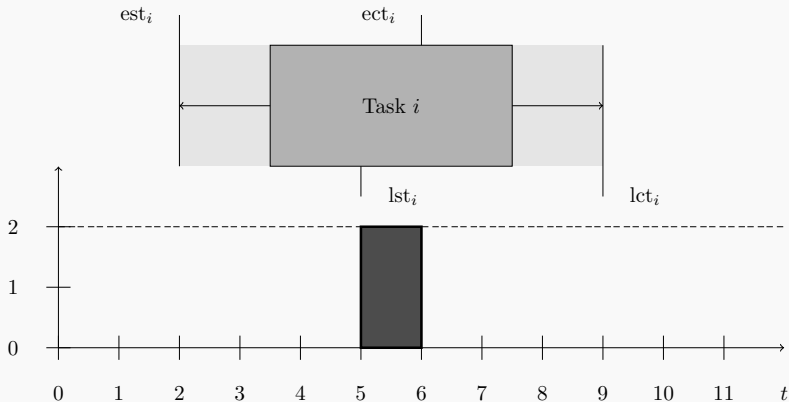


Notation

- est : earliest starting time
- lst : latest starting time
- $ect = est + p$: earliest completion time
- $lct = lst + p$: latest completion time

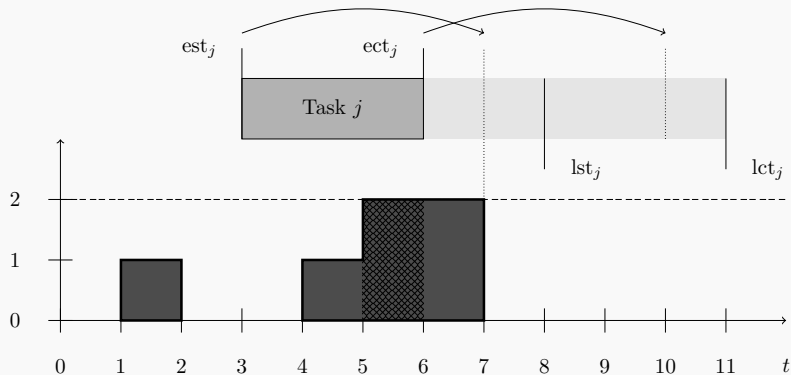
The Time-Tabling rule

The compulsory part is the interval $[lst, ect)$, if non-empty



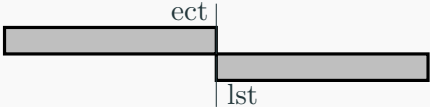
The Time-Tabling rule

The profile (the aggregation of compulsory parts) helps deduce invalid values

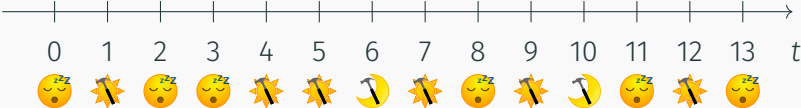


Second Contribution - The CUMULATIVEOVERTIME constraint

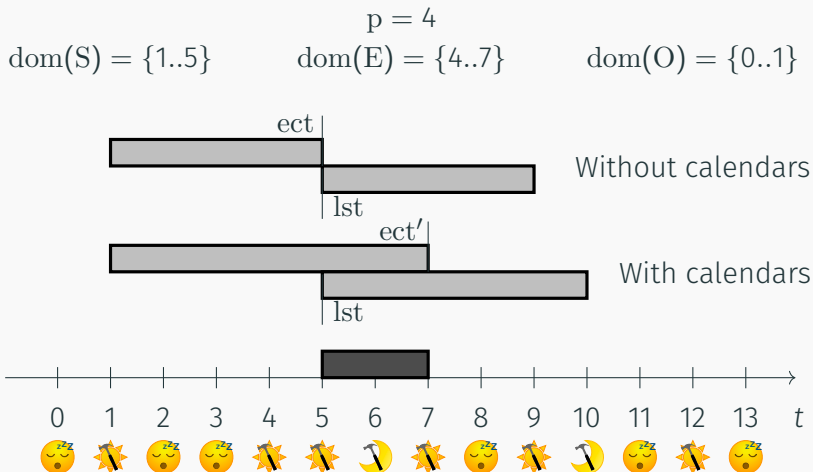
$p = 4$
 $\text{dom}(S) = \{1..5\}$ $\text{dom}(E) = \{4..7\}$ $\text{dom}(O) = \{0..1\}$



Without calendars

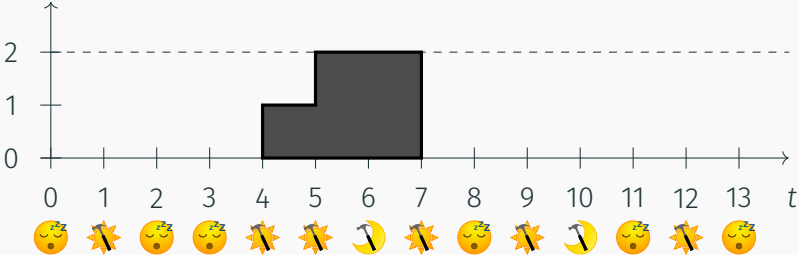


Second Contribution - The CUMULATIVEOVERTIME constraint



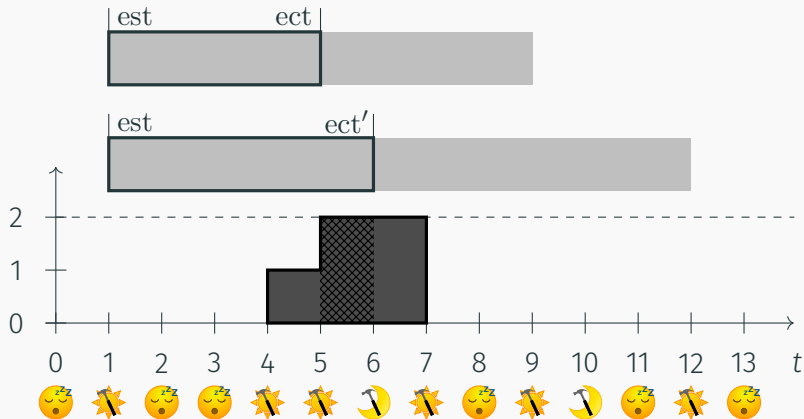
Second Contribution - The CUMULATIVEOVERTIME constraint

$p = 3$
 $\text{dom}(S) = \{1..9\}$ $\text{dom}(E) = \{4..5\}$ $\text{dom}(O) = \{0..1\}$



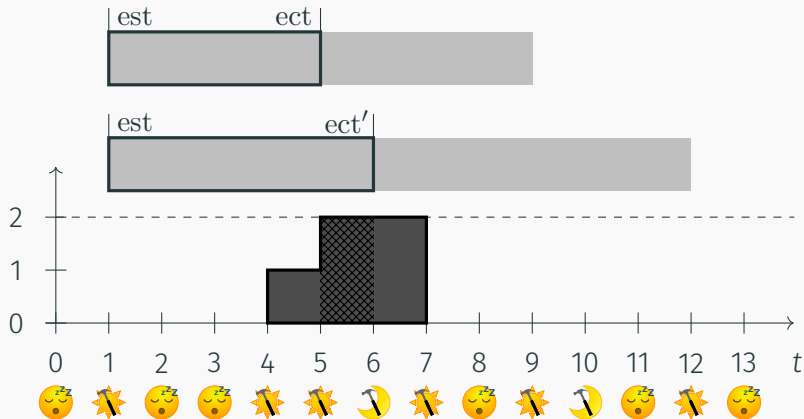
Second Contribution - The CUMULATIVEOVERTIME constraint

$p = 3$
 $\text{dom}(S) = \{1..9\}$ $\text{dom}(E) = \{4..5\}$ $\text{dom}(O) = \{0..1\}$



Second Contribution - The CUMULATIVEOVERTIME constraint

$p = 3$
 $\text{dom}(S) = \{7..9\}$ $\text{dom}(E) = \{4..5\}$ $\text{dom}(O) = \{0..1\}$



Problem

We solve the RCPSP augmented with calendars and overtime.

There are resource, release and deadline, precedence, and calendar constraints.

We minimize overtime costs with a restrained horizon.

Instances

We use instances from PSPLIB, BL, and PACK

They are augmented with pseudo-realistic calendars.

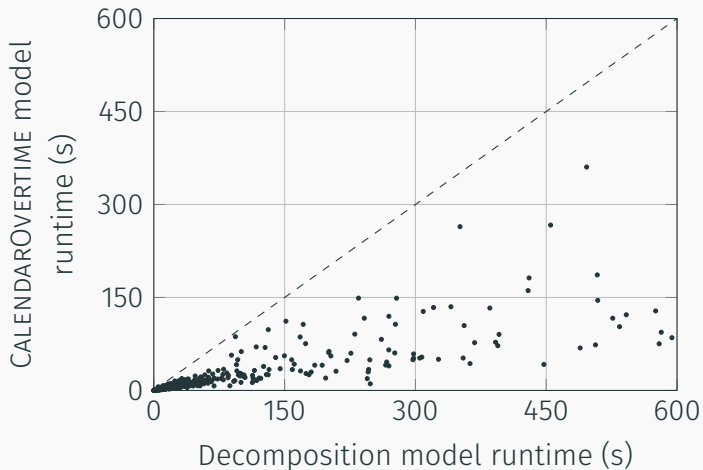
The 3 MiniZinc models

- Decomposition model : Decomposes the calendar constraints into ELEMENT constraints.
- CALENDAROVERTIME model : Uses our CALENDAROVERTIME constraints.
- CUMULATIVEOVERTIME model : Uses both our new constraints.

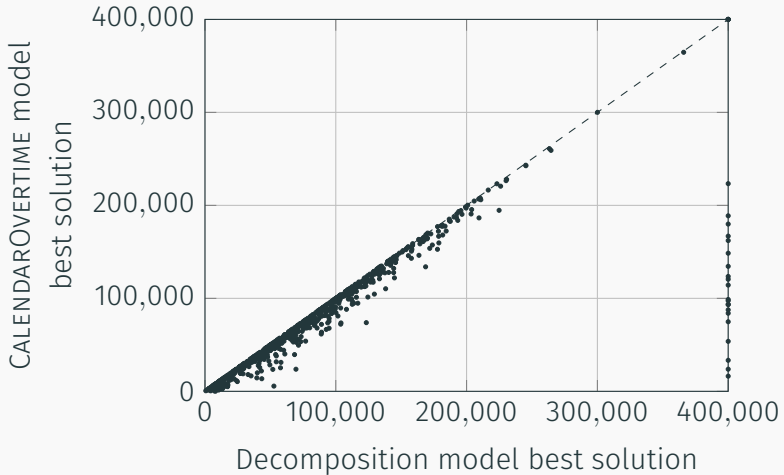
Implementation

Our constraints are implemented in the Chuffed solver (which does lazy clause generation).

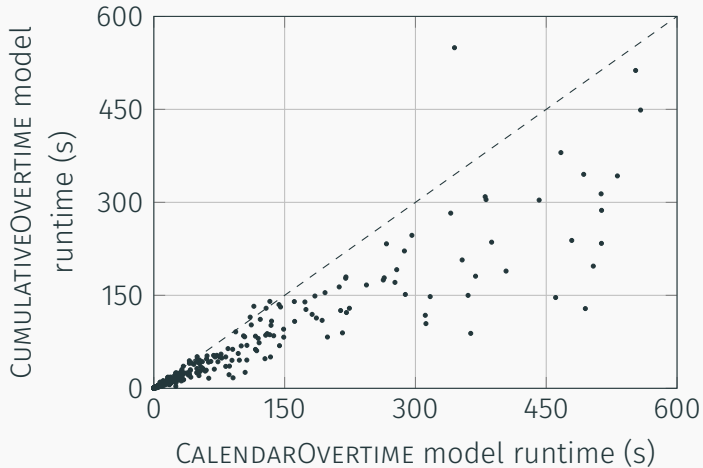
Results - Decomposition vs. CALENDAROVERTIME



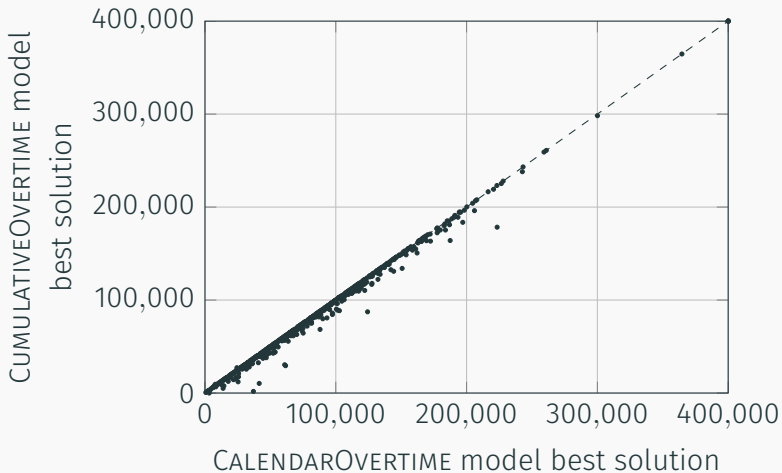
Results - Decomposition vs. CALENDAROVERTIME



Results - CALENDAROVERTIME vs. CUMULATIVEOVERTIME



Results - CALENDAROVERTIME vs. CUMULATIVEOVERTIME



Contributions

- The CALENDAROVERTIME constraint (with a propagator that applies bounds consistency of the S, E and O variables in $\mathcal{O}(|\text{dom}(S)|)$).
- The CUMULATIVEOVERTIME constraint (with a propagator that incorporate calendars in the Time-Tabling rule).

Advantages of the new constraints

- Make modeling with calendars simpler
- Lead to better resolutions

The new constraints could be used in other problems than the RCPSP augmented with calendars.

Thank you for your attention

Link to the code, instances, and models :

