# Efficient Implementation of the Global Cardinality Constraint with Costs

**Margaux SCHMIED**, Université Côte d'Azur
Jean-Charles RÉGIN, Université Côte d'Azur

i3S sophia antipolis

3iA Côte d'Azur
Institut interdisciplinaire d'intelligence artificielle

UNIVERSITÉ CÔTE D'AZUR

# Plan

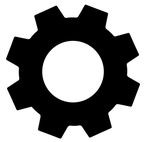
- Assignment problems with costs
- Global Cardinality Constraint with Cost
- Our approach
- Results
- Conclusion

# Assignment problems with costs

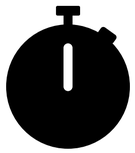
# Assignment problems with costs

We can assign:

An activity for a child according to his preferences



An object to a machine according to production cost



A task to an employee according to the time taken to complete it

# Assignment problems with costs

Other forms of assignment:

Scheduling

Vehicle routing

Travelling salesman problem

# Assignment problems with costs

Peter

Paul

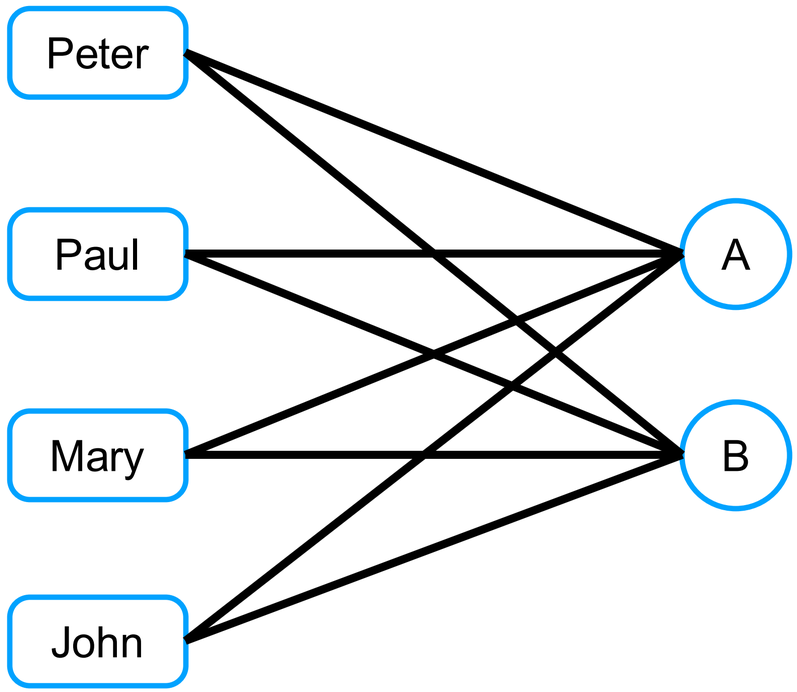
Mary

John

A

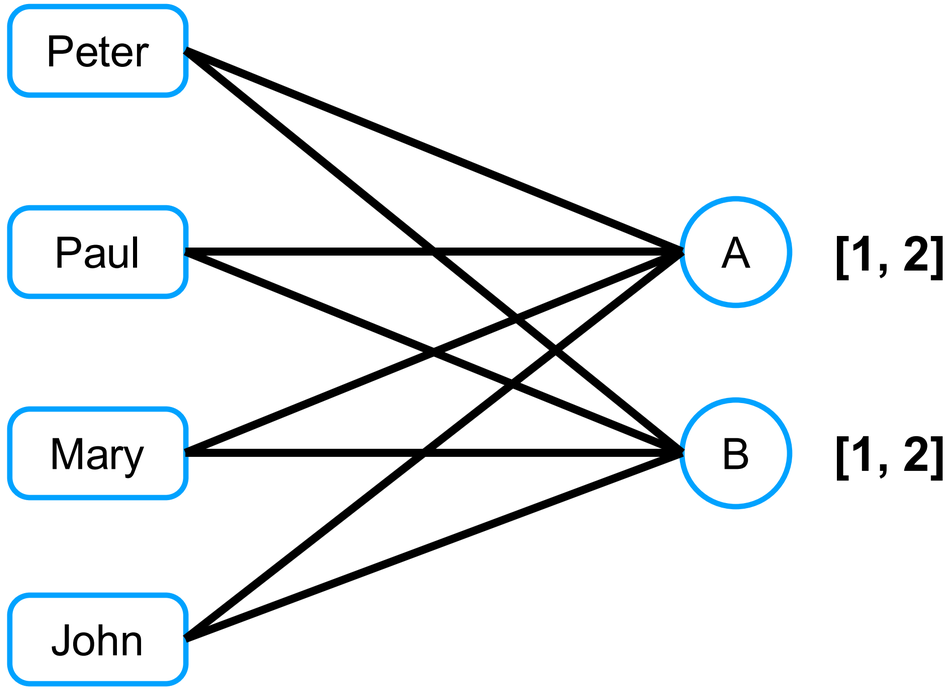
B

# Assignment problems with costs




Ability to perform the task

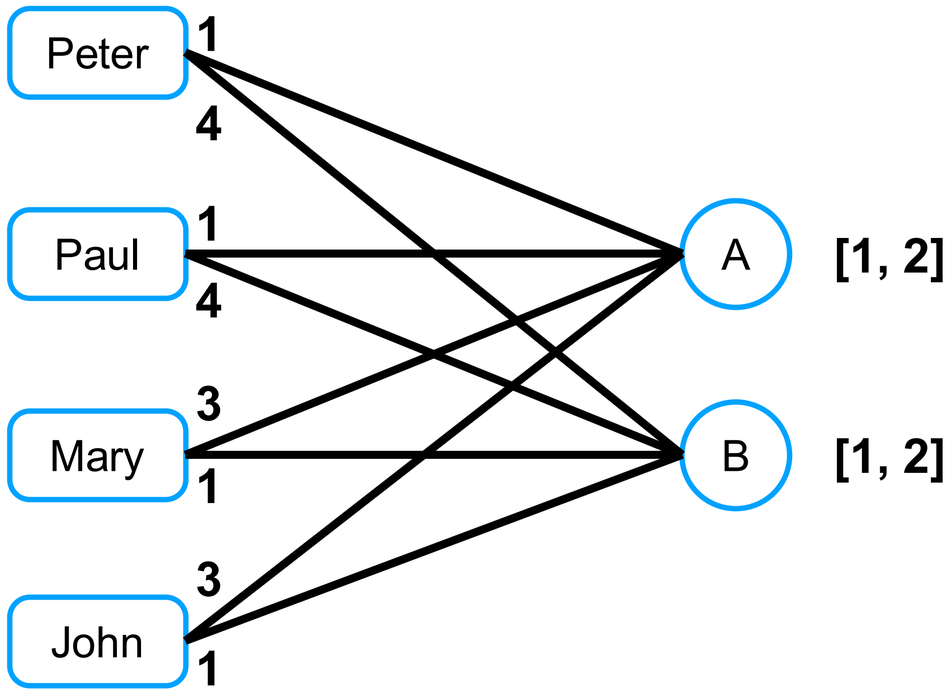
# Assignment problems with costs




Ability to perform the task

Number of tasks completed

# Assignment problems with costs

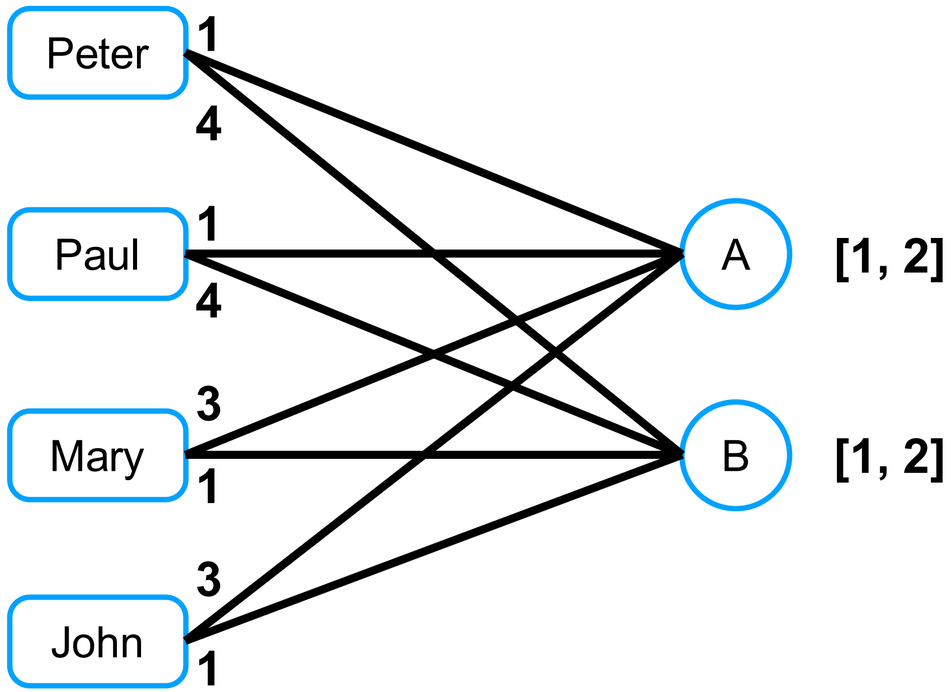



Ability to perform the task

Number of tasks completed

Task completion time

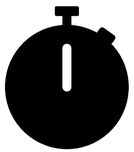
# Assignment problems with costs




Ability to perform the task

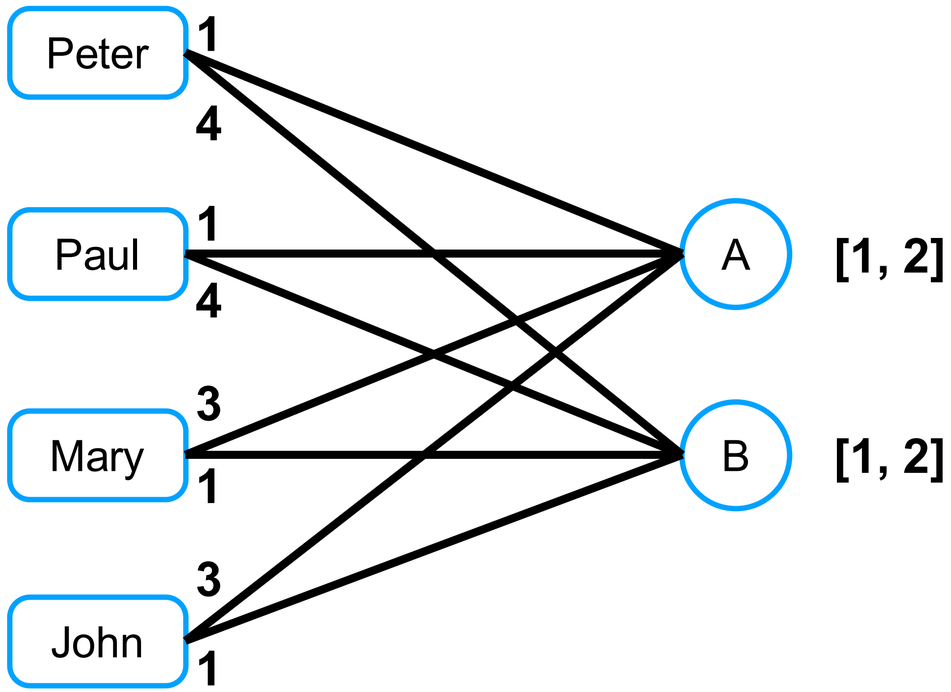
Number of tasks completed

Task completion time



Total authorised time less than 7

# Assignment problems with costs




Ability to perform the task

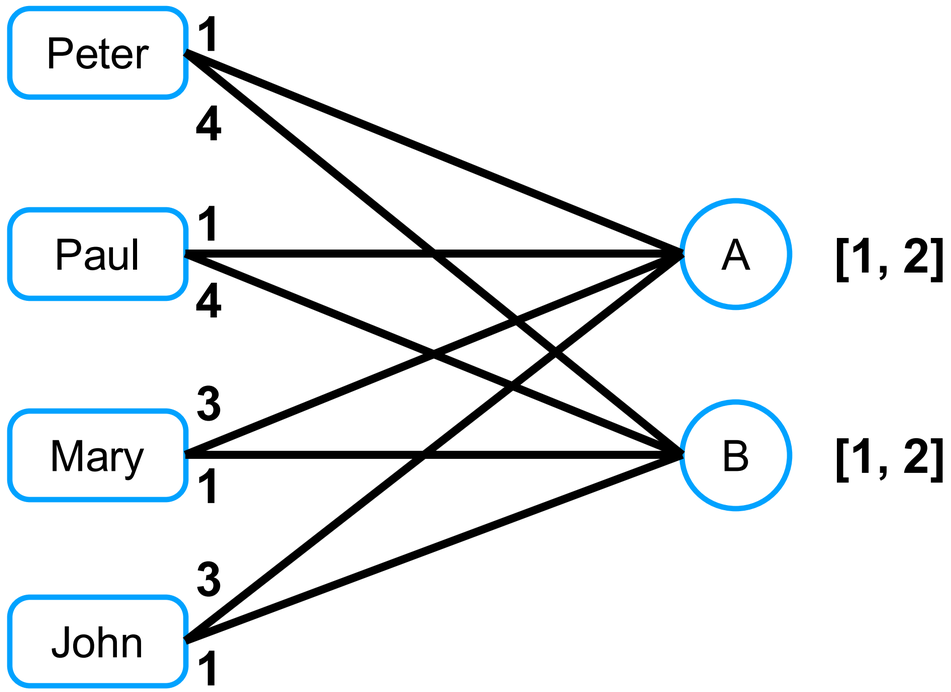
Number of tasks completed

Task completion time

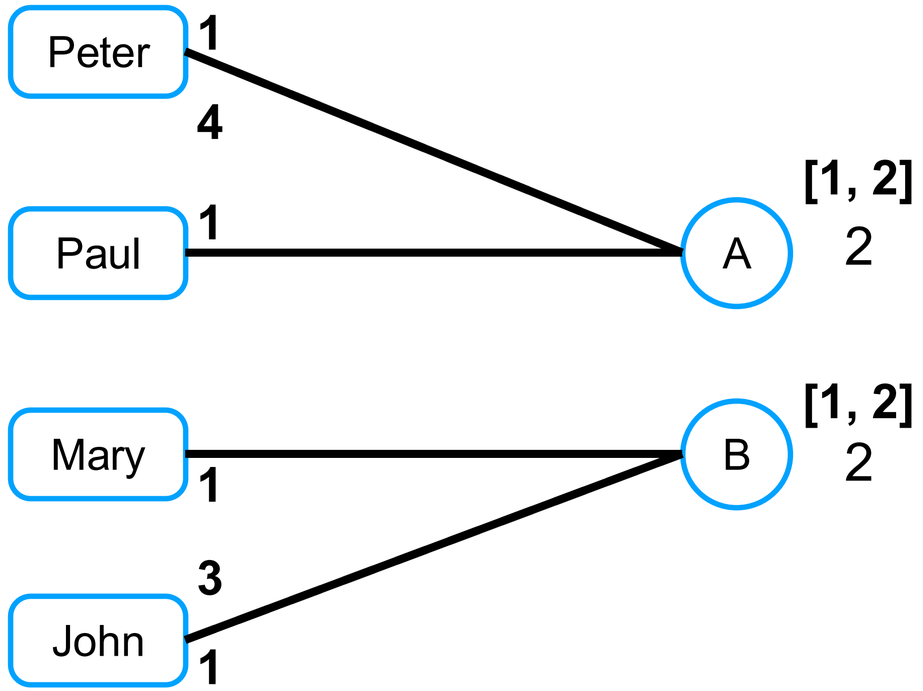
H

Total authorised time less than 7

# Assignment problems with costs




Total time = 4 ≤ 7



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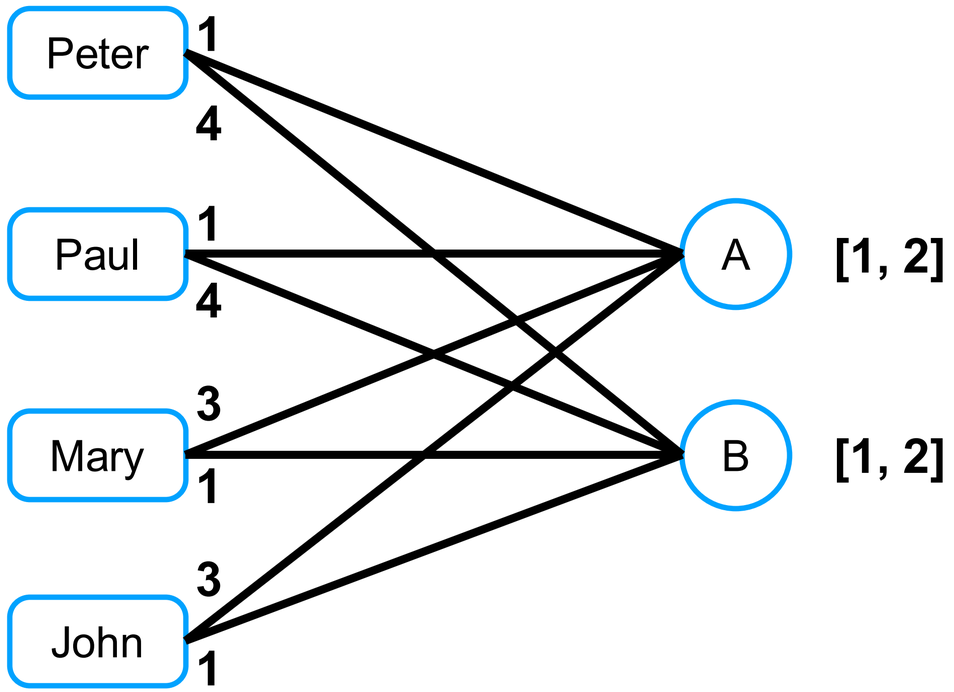


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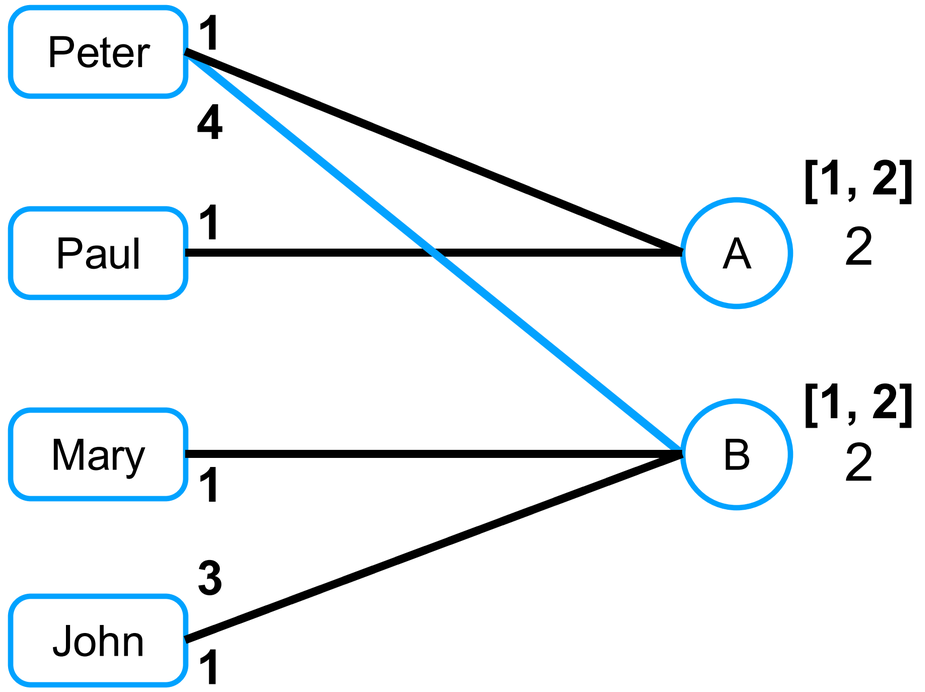

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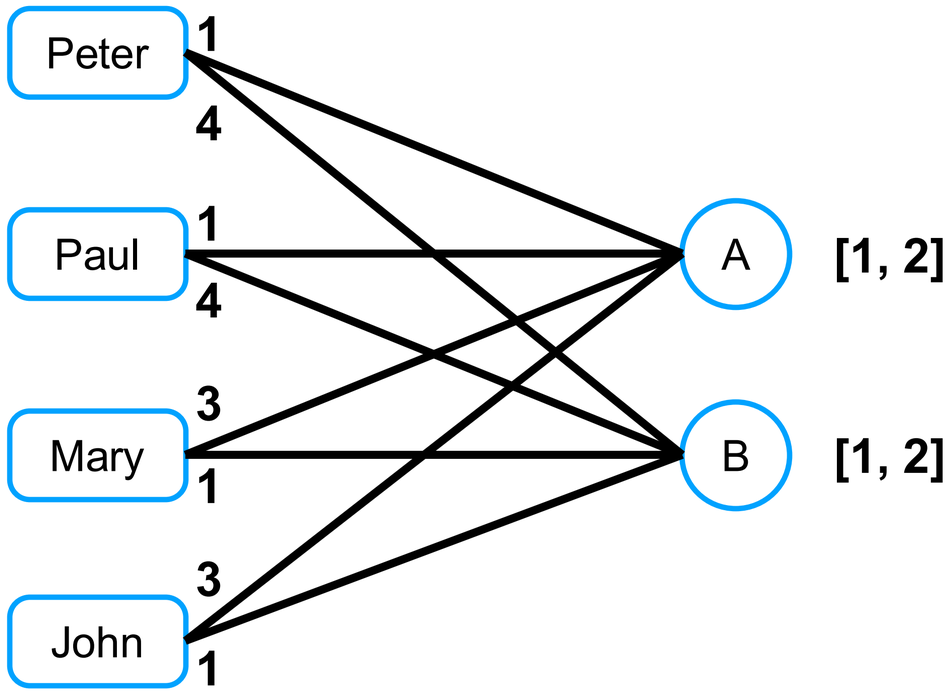


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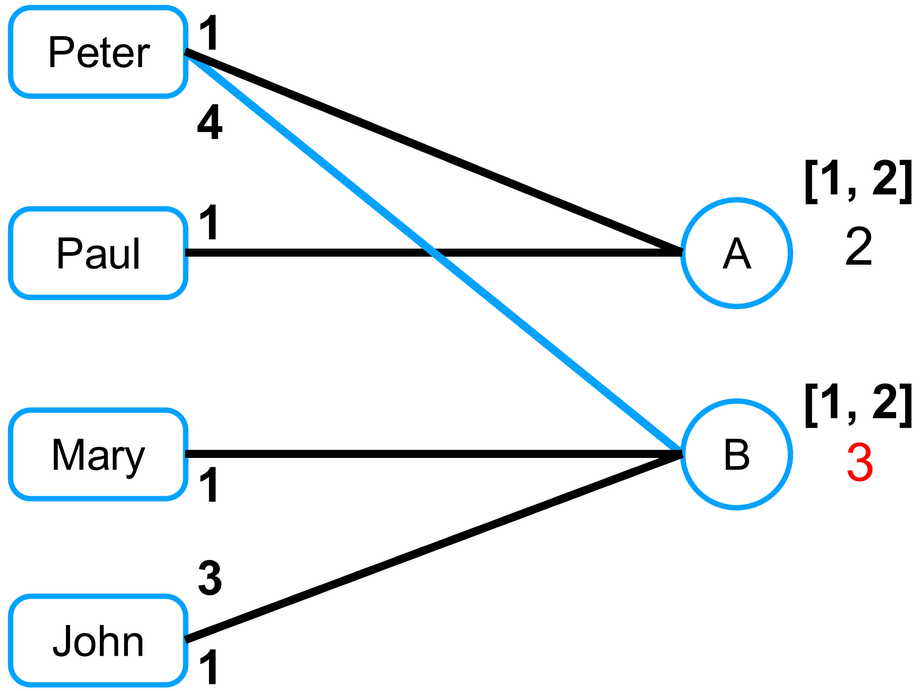

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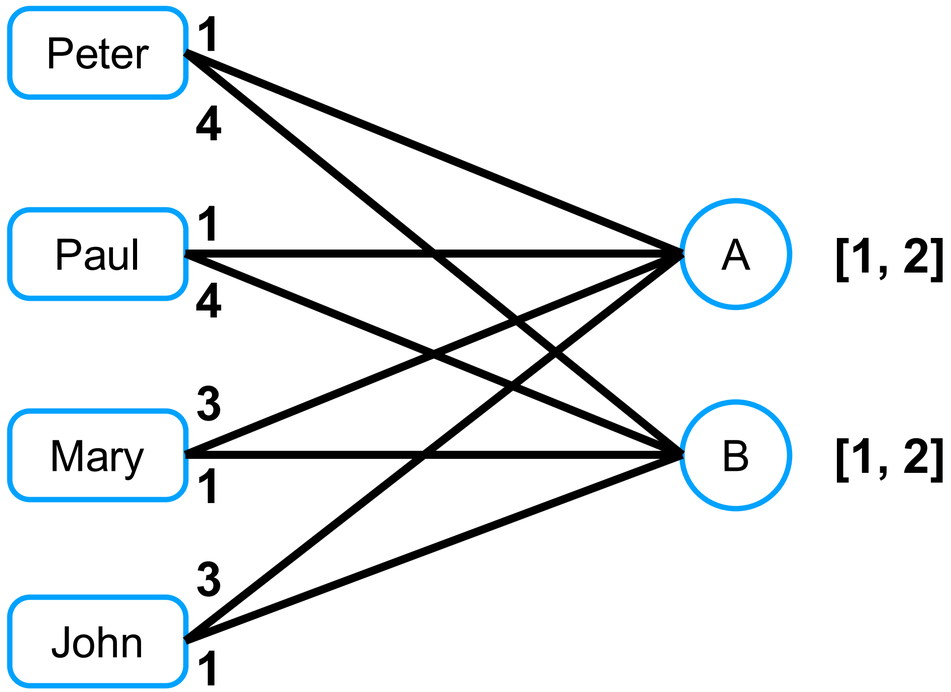


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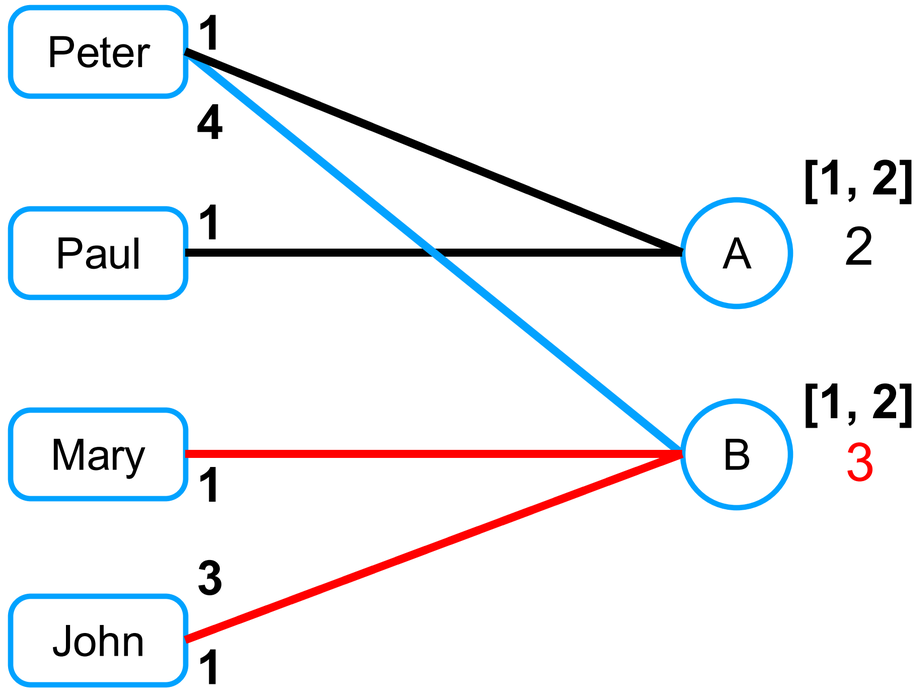

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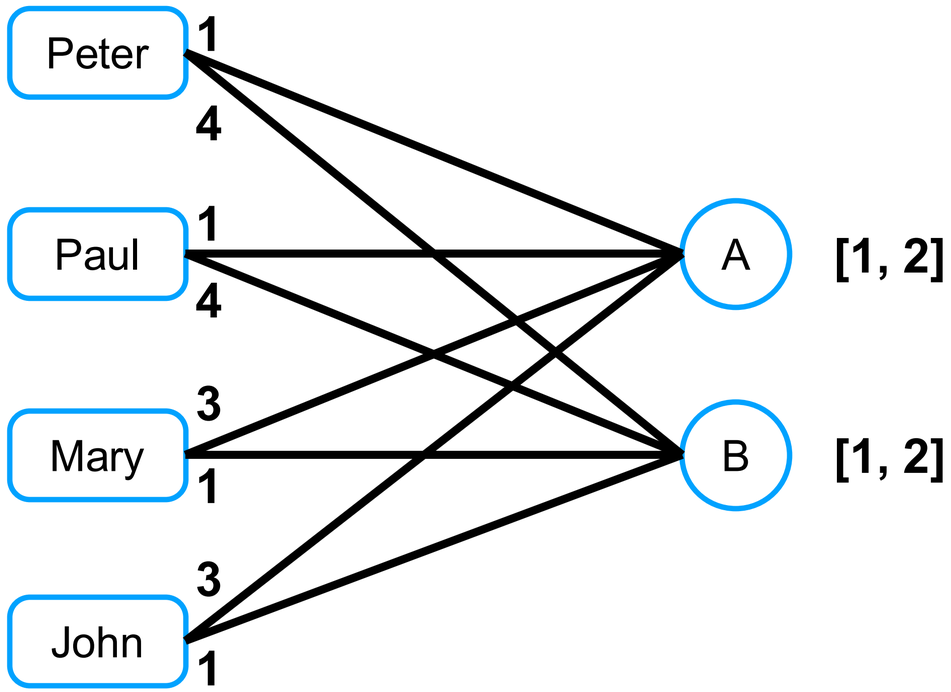


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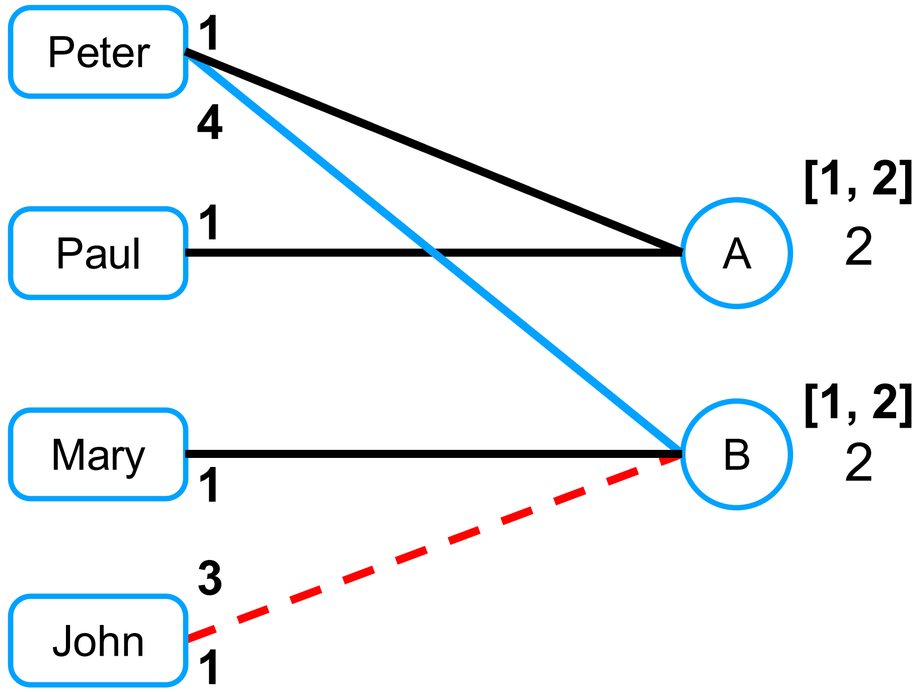

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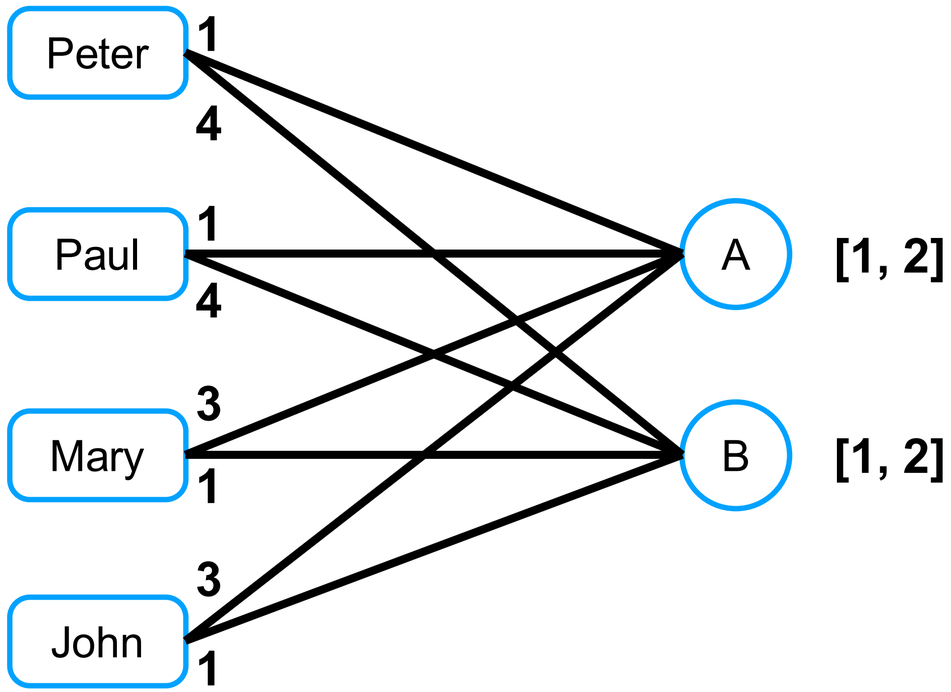


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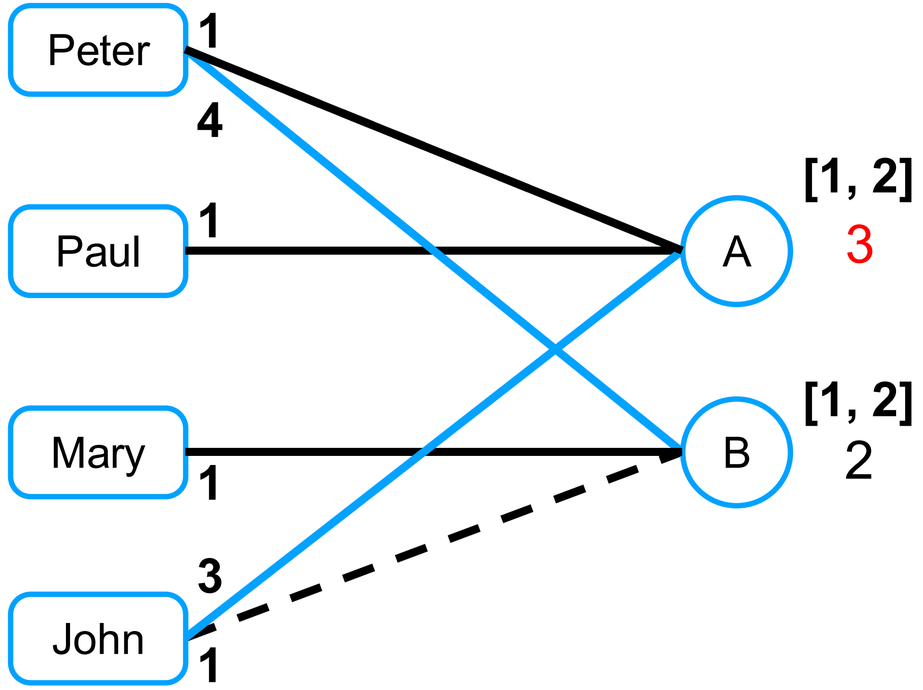

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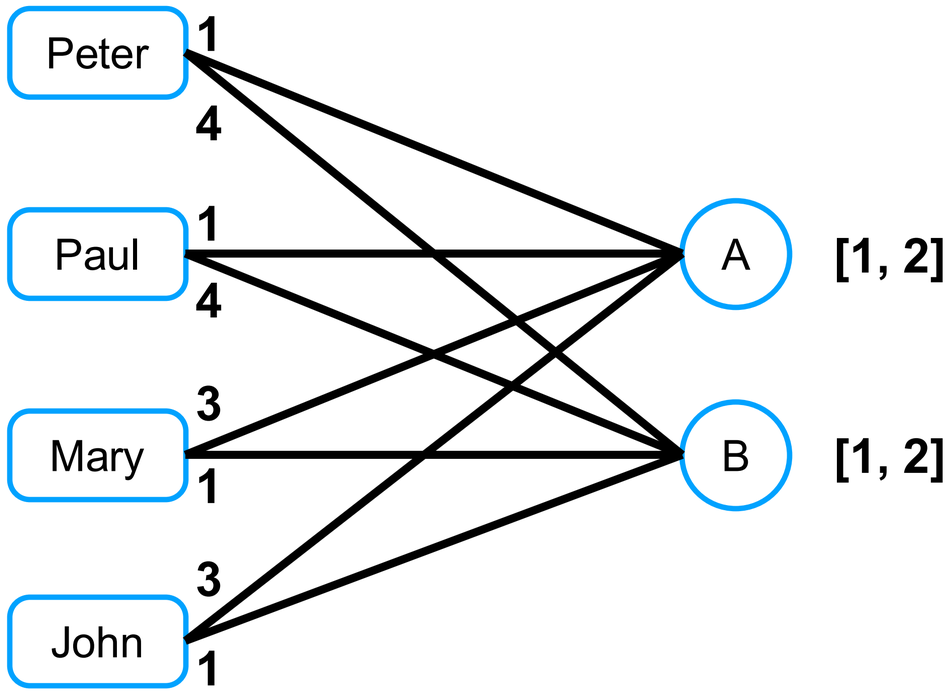


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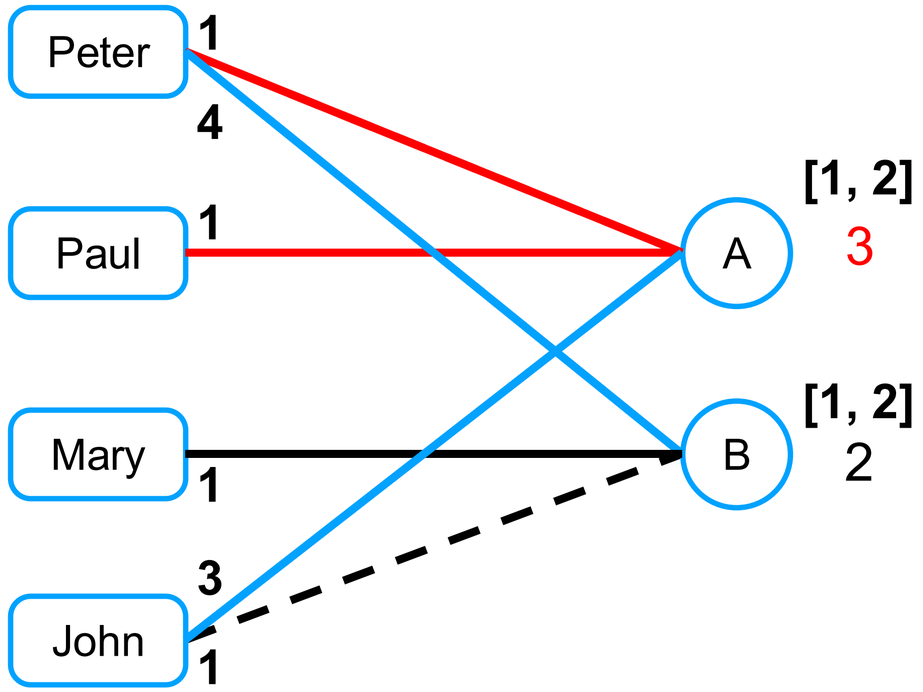

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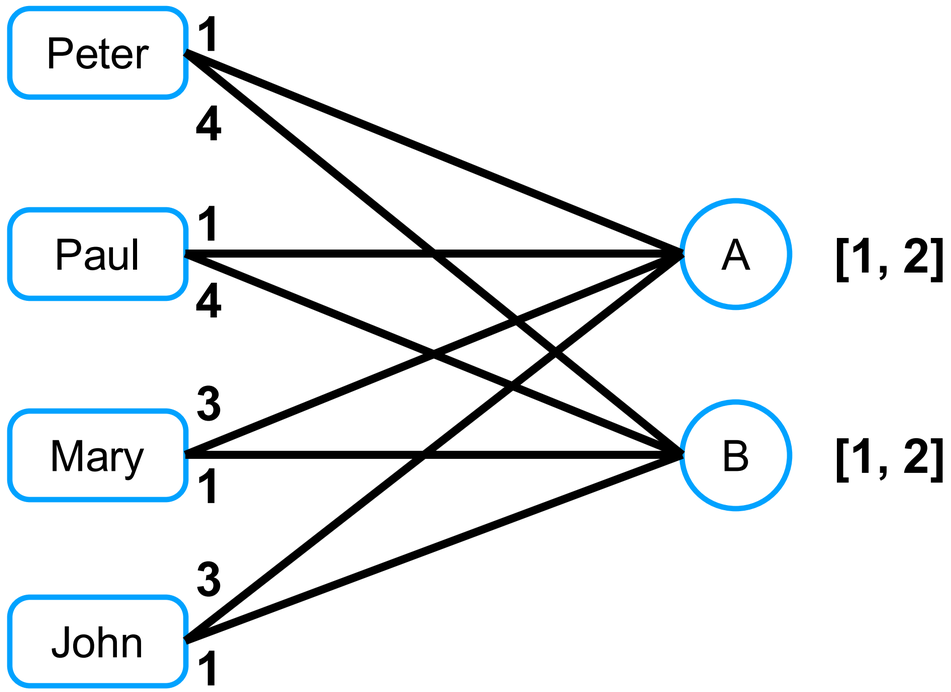


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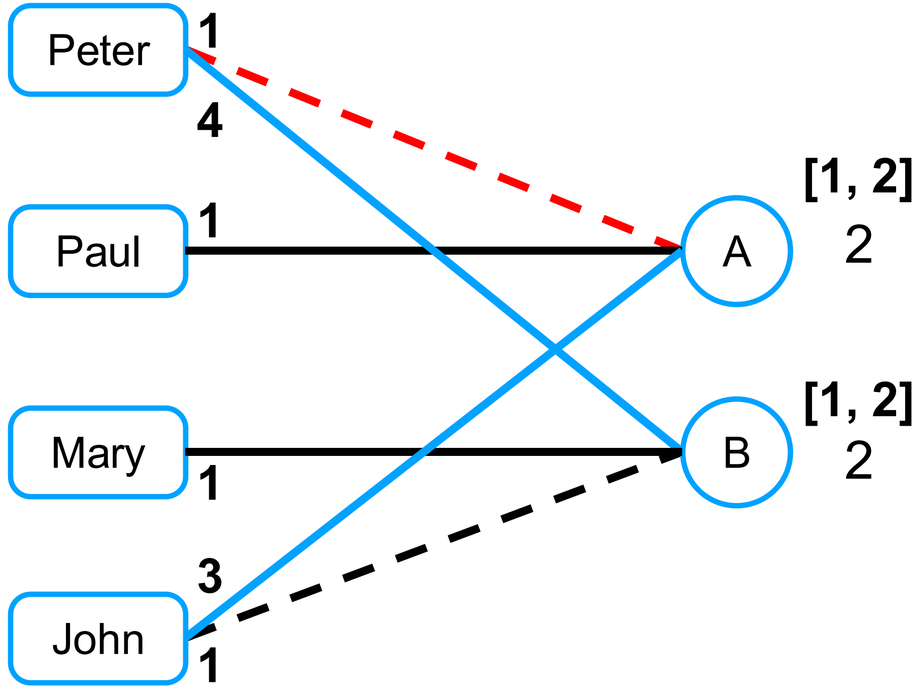

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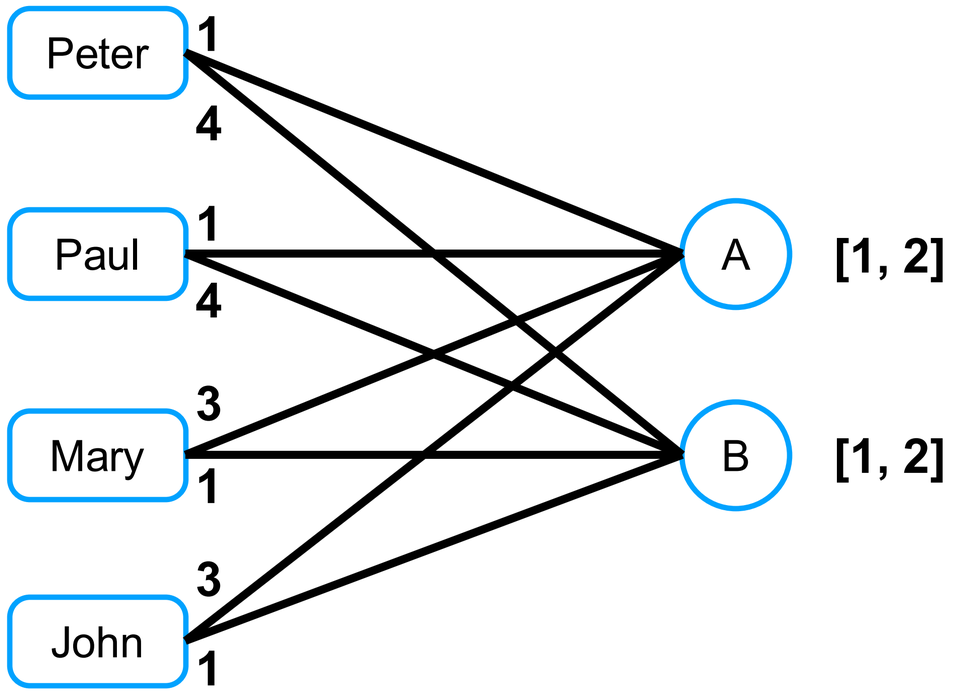
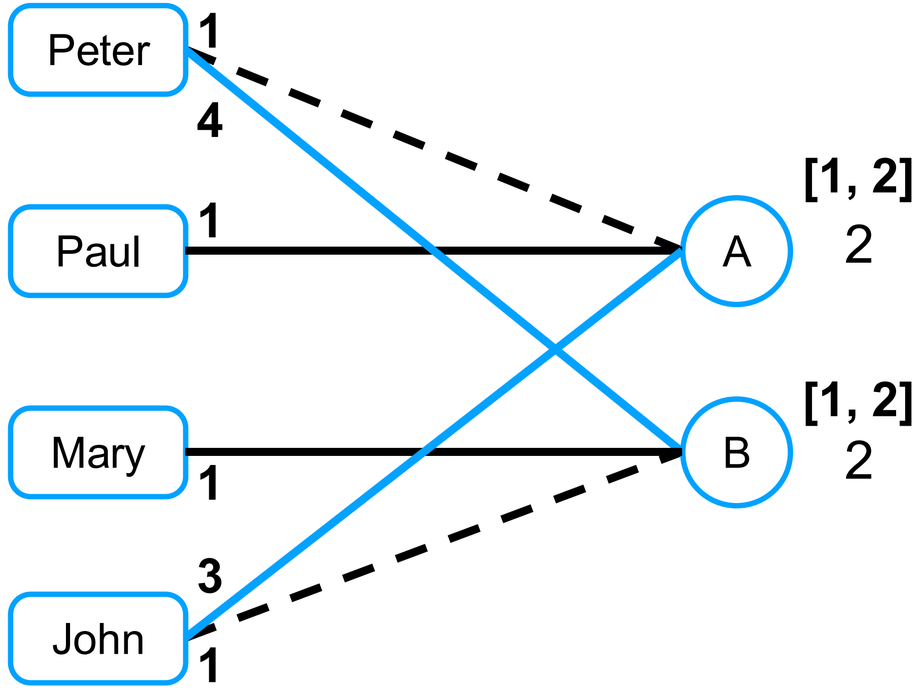



Total time = 9 ≤ 7



# Global Cardinality Constraint with Cost

**[Régin, 2002]**

Efficient Implementation of the Global Cardinality Constraint with Costs

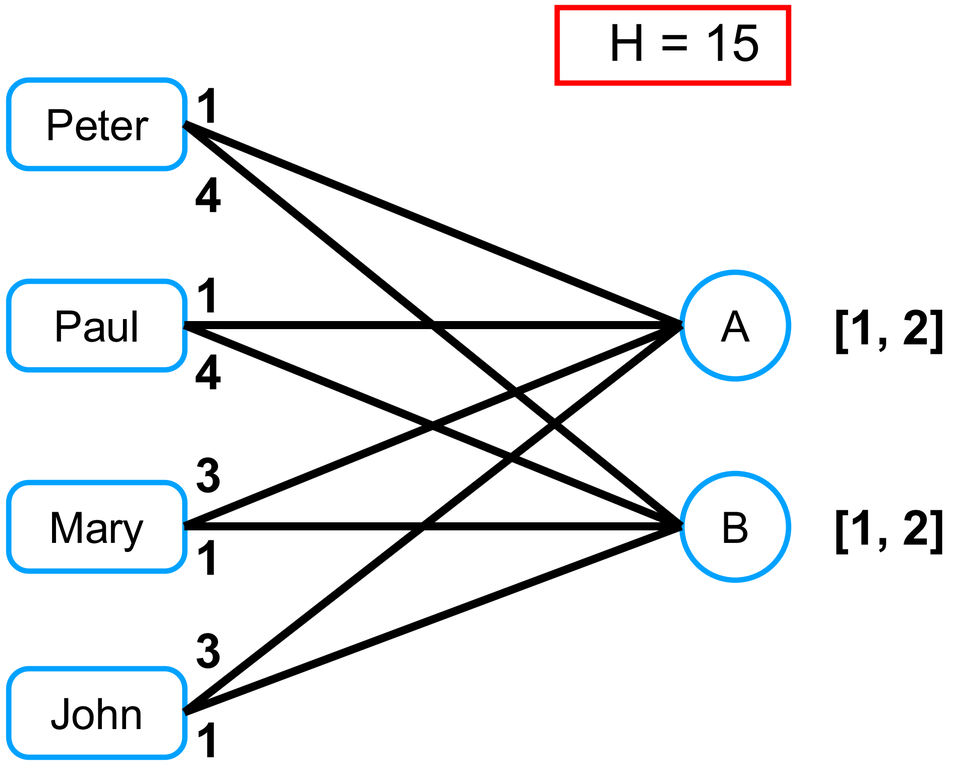
# Global Cardinality Constraint with Cost

1. Is there a solution to our problem?

2. How can we remove assignments that cannot be part of a solution?

# Global Cardinality Constraint with Cost

**1** Is there a solution to our problem?




**Note :**

All assignments can be part of a solution

# Global Cardinality Constraint with Cost

**1** Is there a solution to our problem?

## Finding a solution

- Create the value graph
- Apply the minimum cost maximum flow algorithm

# Global Cardinality Constraint with Cost

**1** Is there a solution to our problem?

## Finding a solution

- Create the value graph
- Apply the minimum cost maximum flow algorithm

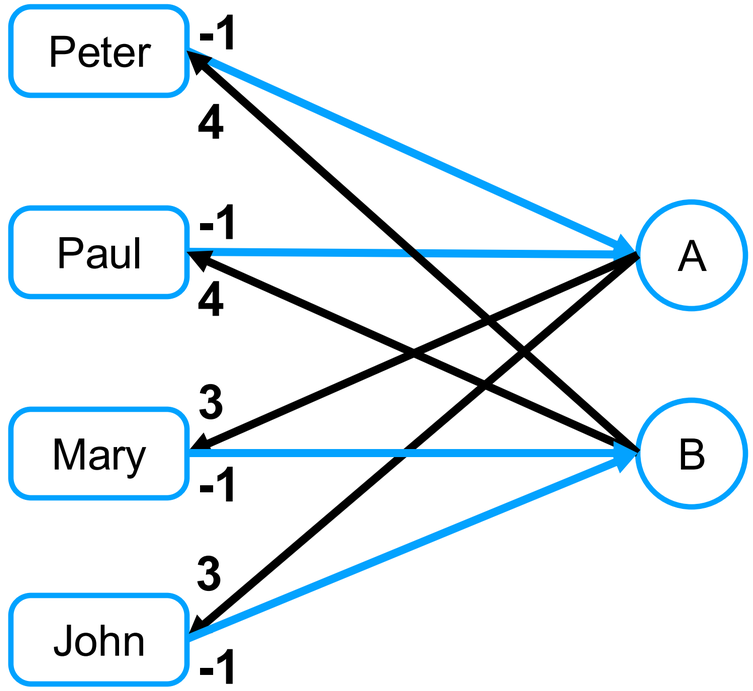
H > optimal cost

15 > 4

Peter = Paul = A

Mary = John = B

Output

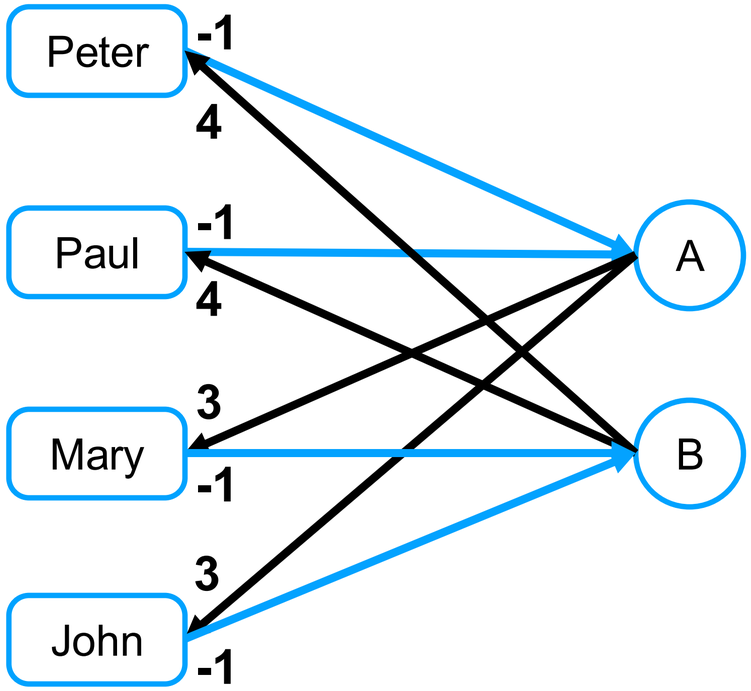


# Global Cardinality Constraint with Cost

**2** How can we remove assignments that cannot be part of a solution?

A value **a** of a variable **x** can belong to a solution IFF:

- (**a**, **x**) belongs to the optimal solution or
- dist(**x**, **a**) ≤ H - optCost - cost(**a**, **x**)

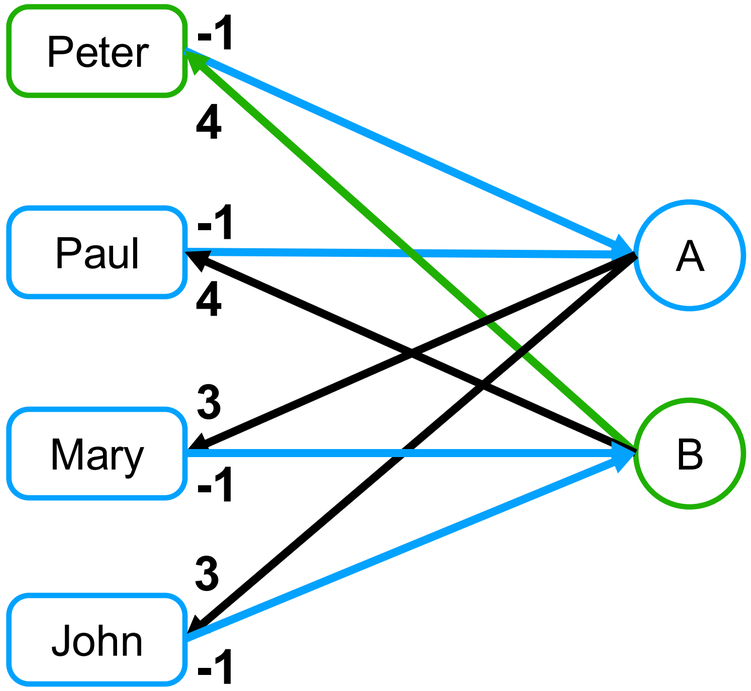


# Global Cardinality Constraint with Cost

**2** How can we remove assignments that cannot be part of a solution?

A value **B** of a variable **Peter** can belong to a solution IFF:

- (**B**, **Peter**) belongs to the optimal solution or
- dist(**Peter**, **B**) ≤ H - optCost - cost(**B**, **Peter**)



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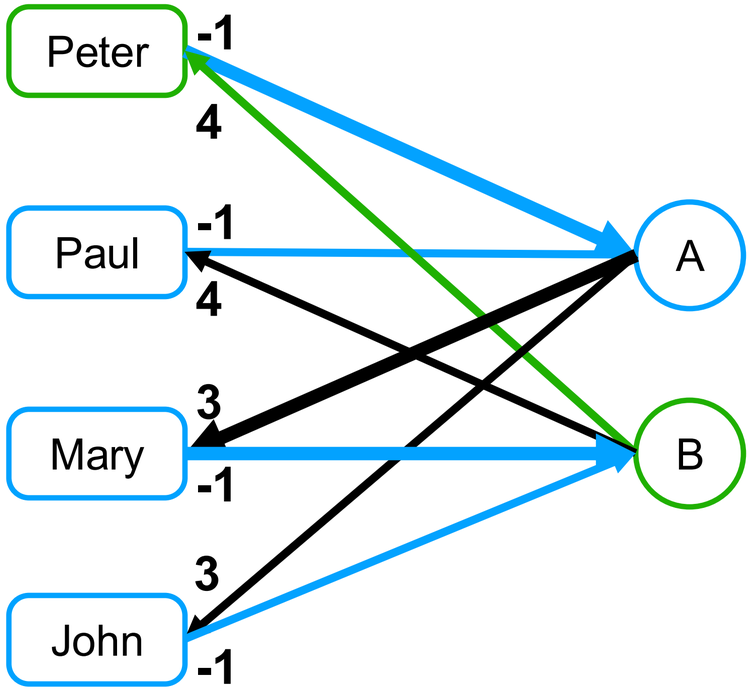
dist(**Peter**, **B**) = 1



# Global Cardinality Constraint with Cost

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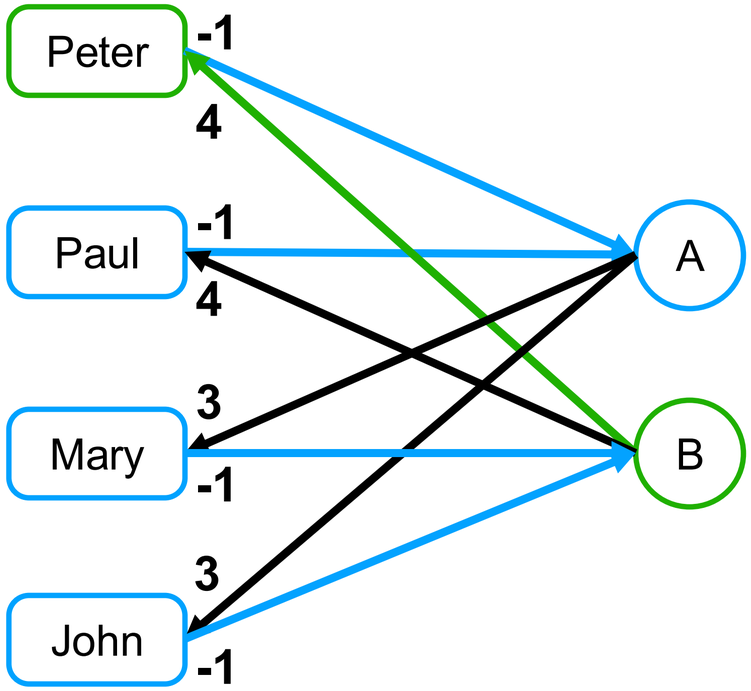
dist(**Peter**, **B**) = 1
H – optCost - cost(**B**, **Peter**) = 15 - 4 - 1 = 10



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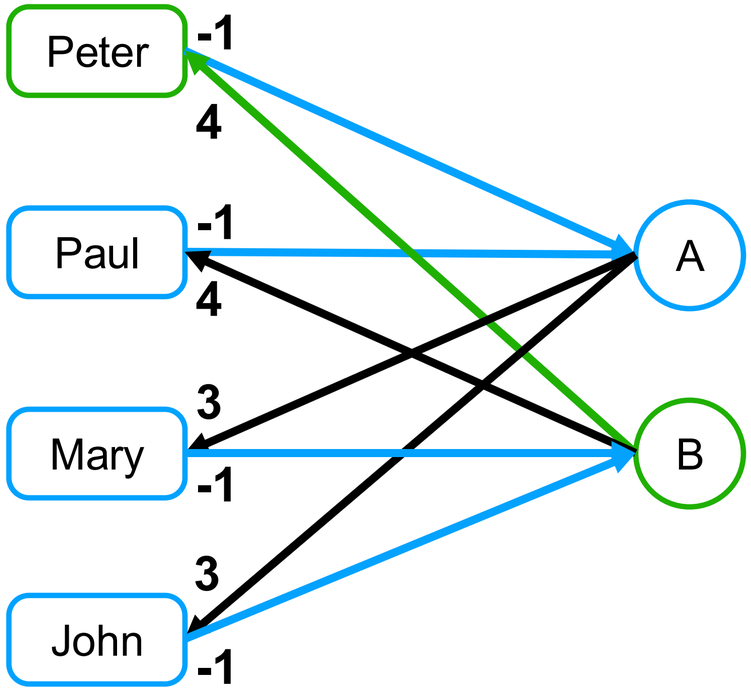


# Our approach

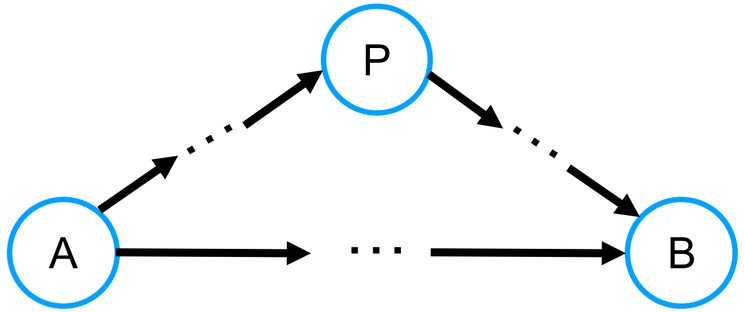
**Is it necessary to compute so many shortest paths?**

# Our approach

Triangular inequality on shortest paths

**Triangular inequality:**

$$dist(A, B) \leq dist(A, P) + dist(P, B)$$



# Our approach

Triangular inequality on shortest paths

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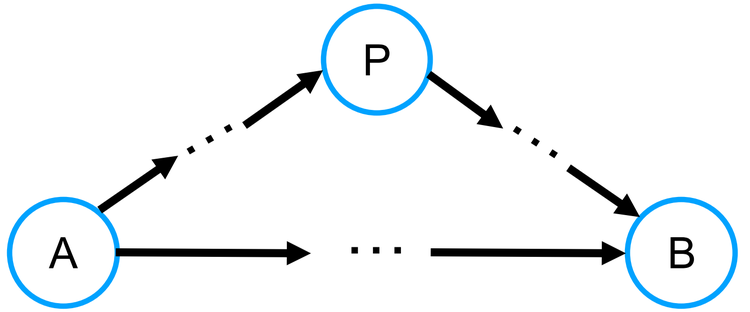
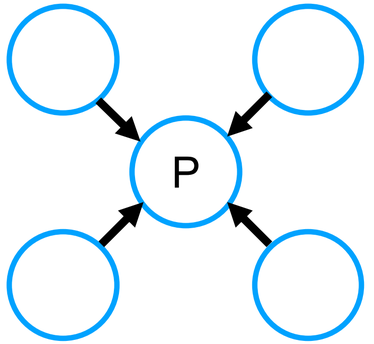
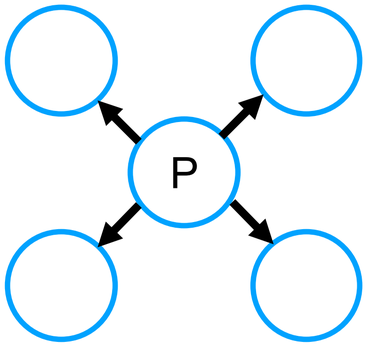



If we know the shortest paths from:

- All nodes to P
- P to all nodes

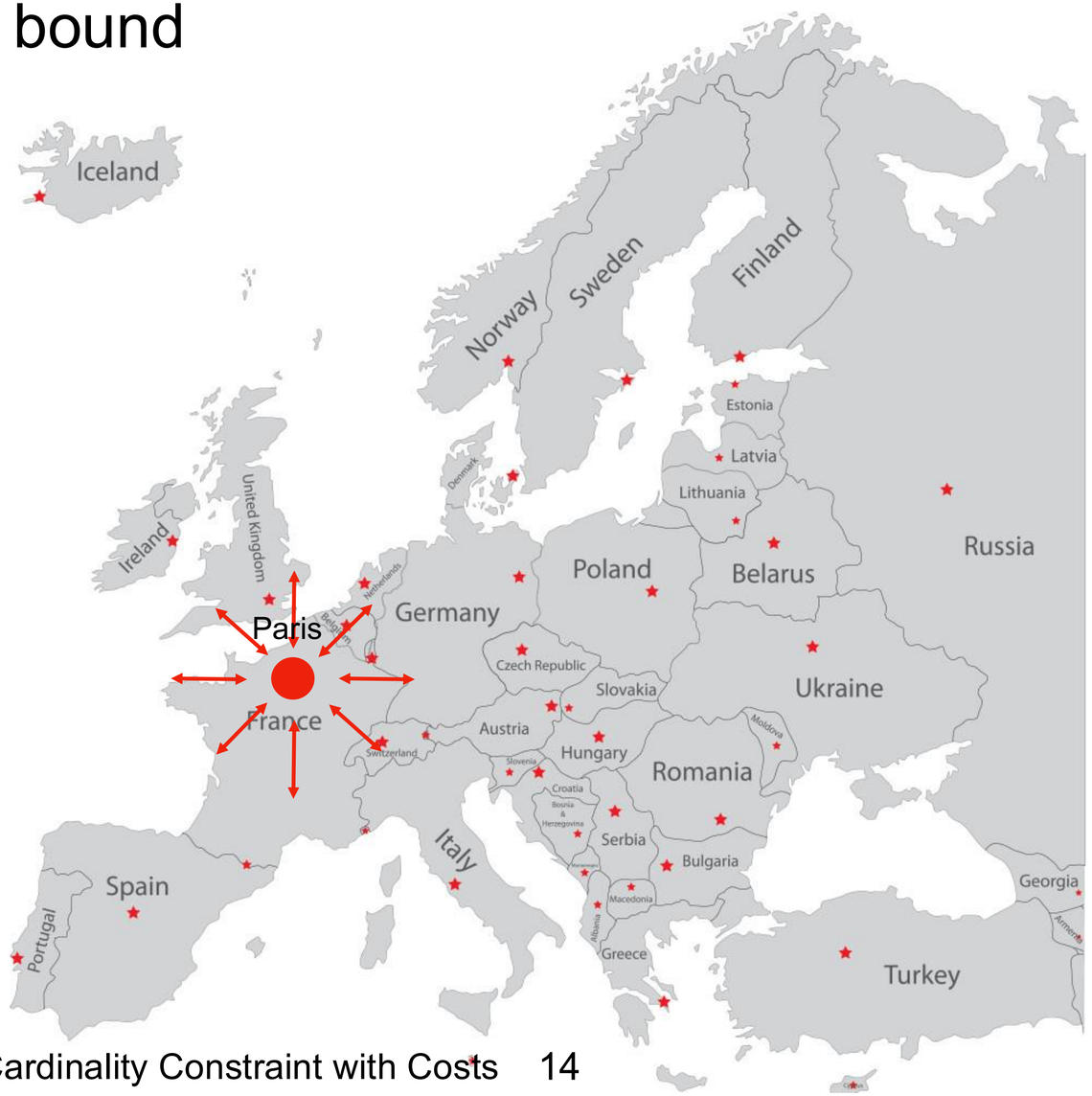
dist(A, P) + dist(P, B) is in O(1)



# Our approach

Shortest path upper bound



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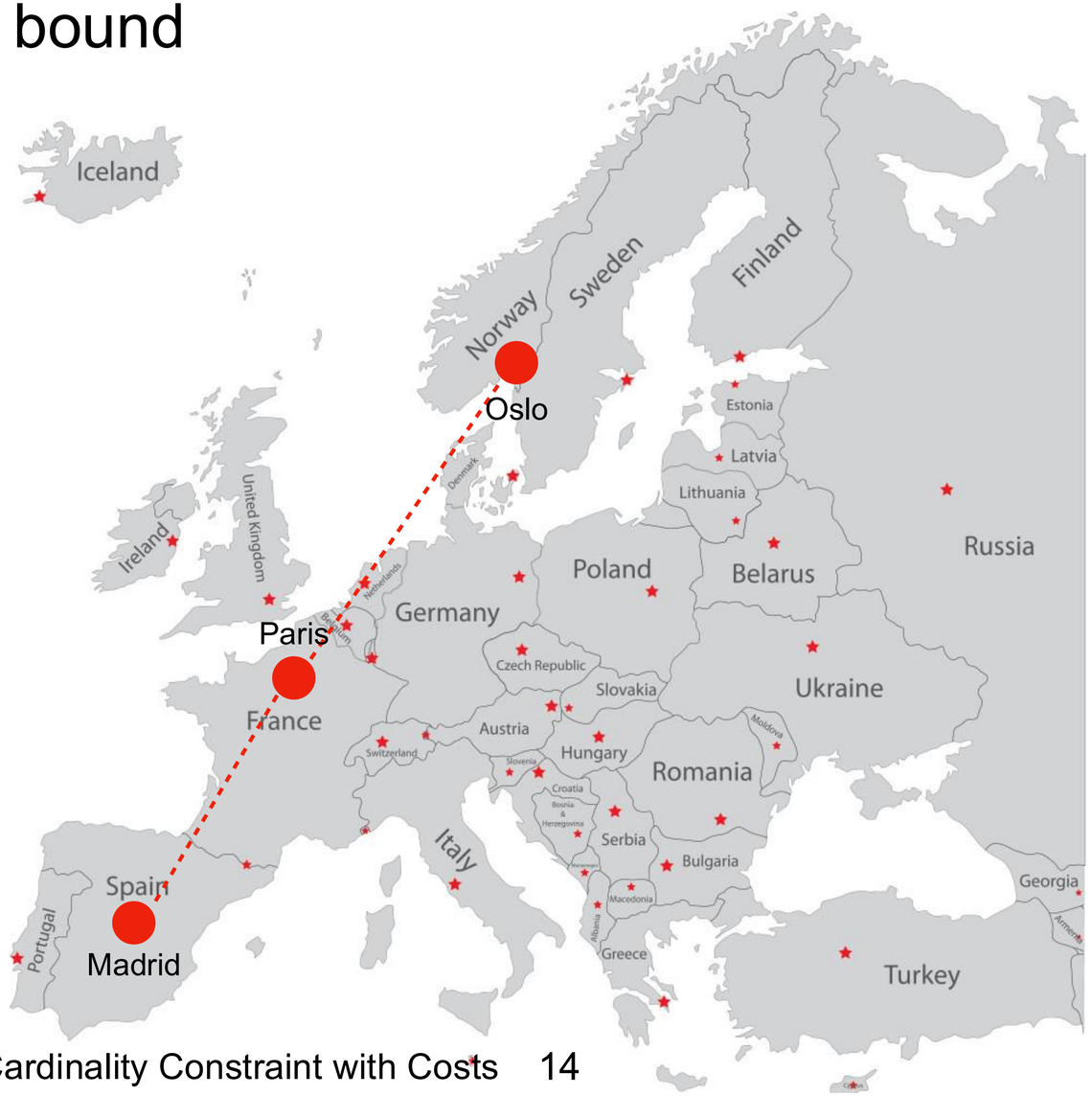

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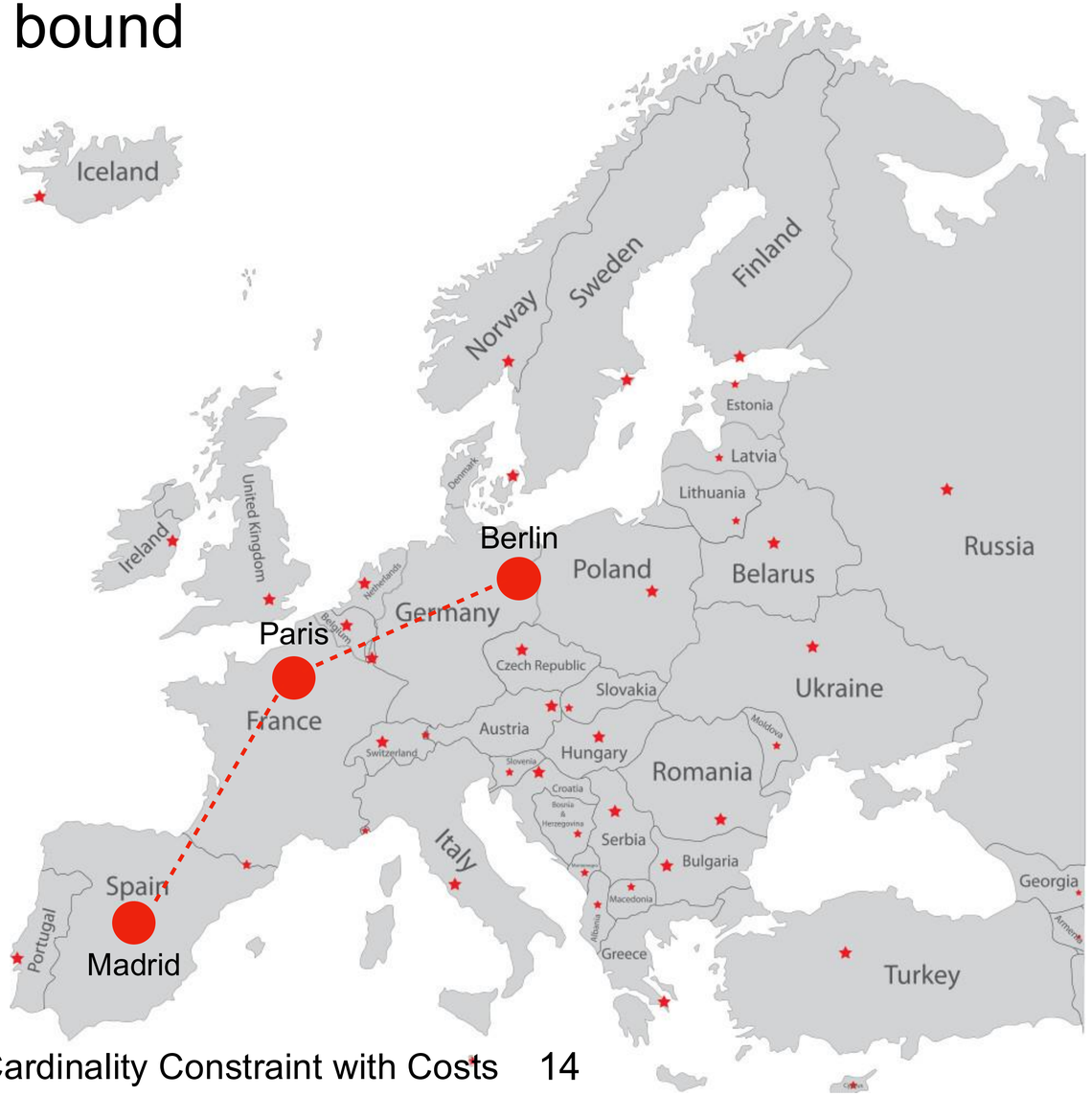

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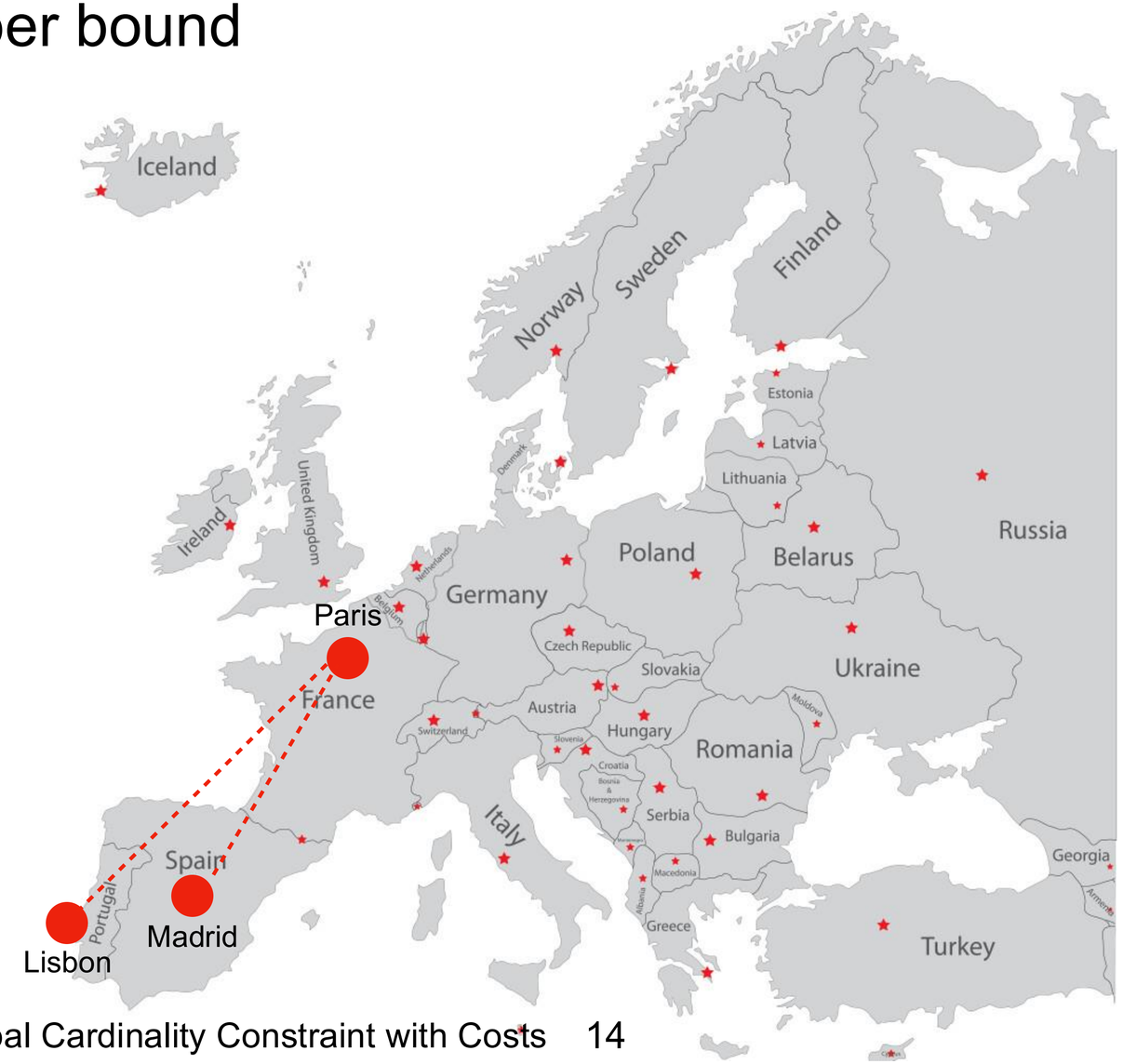
Shortest path upper bound



# Our approach

Triangular inequality:

$dist(A, B) \leq dist(A, P) + dist(P, B)$

# Our approach

Triangular inequality:

dist(A, B) ≤ dist(A, P) + dist(P, B)

A value **a** of a variable **x** belongs to a solution IFF:

- (**a**, **x**) belongs to the optimal solution or
- dist(**x**, **a**) ≤ H - optCost - cost(**a**, **x**)

# Our approach

Triangular inequality:

dist(A, B) ≤ dist(A, P) + dist(P, B)

A value **a** of a variable **x** belongs to a solution IFF:

- (**a**, **x**) belongs to the optimal solution or
- dist(**x**, **a**) ≤ H - optCost - cost(**a**, **x**)

Given a landmark **P**, a value **a** of a variable **x** if:

- dist(**x**, **P**) + dist(**P**, **a**) ≤ H - optCost - cost(**a**, **x**)

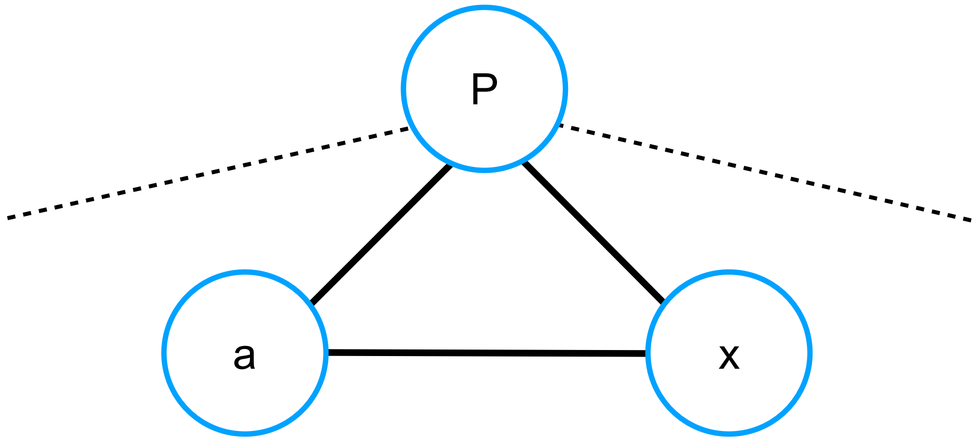
Then **a** belongs to a solution

# Our approach

Given a landmark **P**, a value **a** of a variable **x** if:

dist(**x**, **P**) + dist(**P**, **a**) ≤ H - optCost - cost(**a**, **x**)

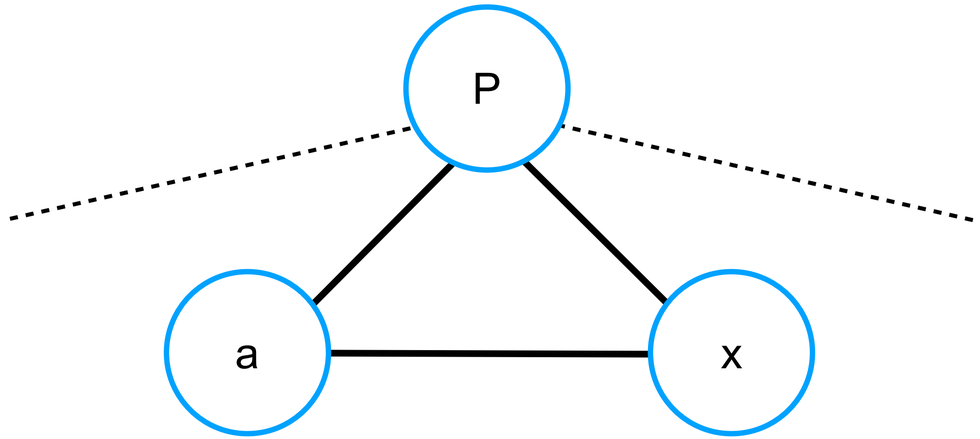
Then **a** belongs to a solution




Note :

- dist(**a**, **x**) ≤ dist(**a**, **P**) + dist(**P**, **x**)
- dist(**a**, **P**) + dist(**P**, **x**) ≤ $dist_{max}(*, \mathbf{P}) + dist_{max}(\mathbf{P}, *)$

# Our approach




Let **P** be a landmark if:

- $dist_{max}(*, \mathbf{P}) + dist_{max}(\mathbf{P}, *) \leq H - optCost - costmax$

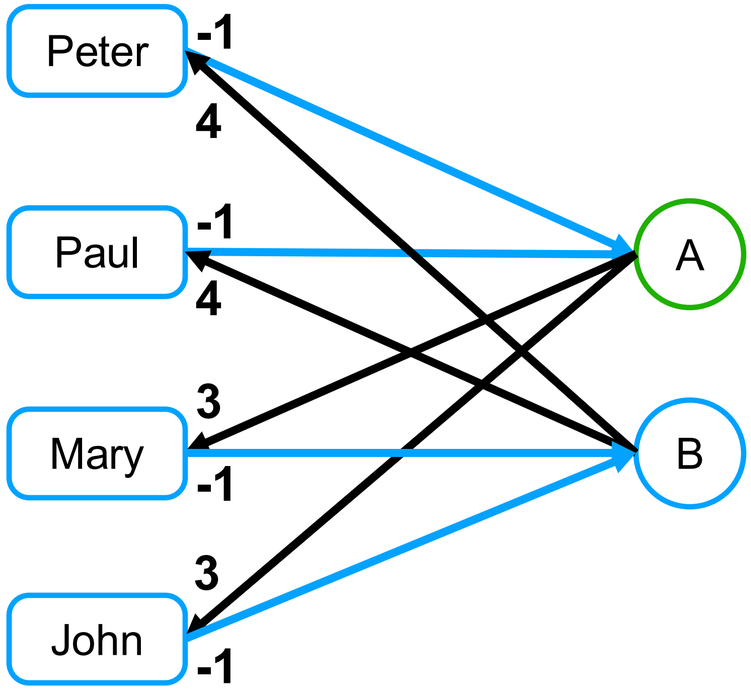
Then all values **a** of all variable **x** belong to a solution

# Our approach

Let **P** be a landmark if:

- distmax(∗, **P**) + distmax(**P**, ∗) ≤ H - optCost - costmax

Then all values **a** of a variable **x** belong to a solution

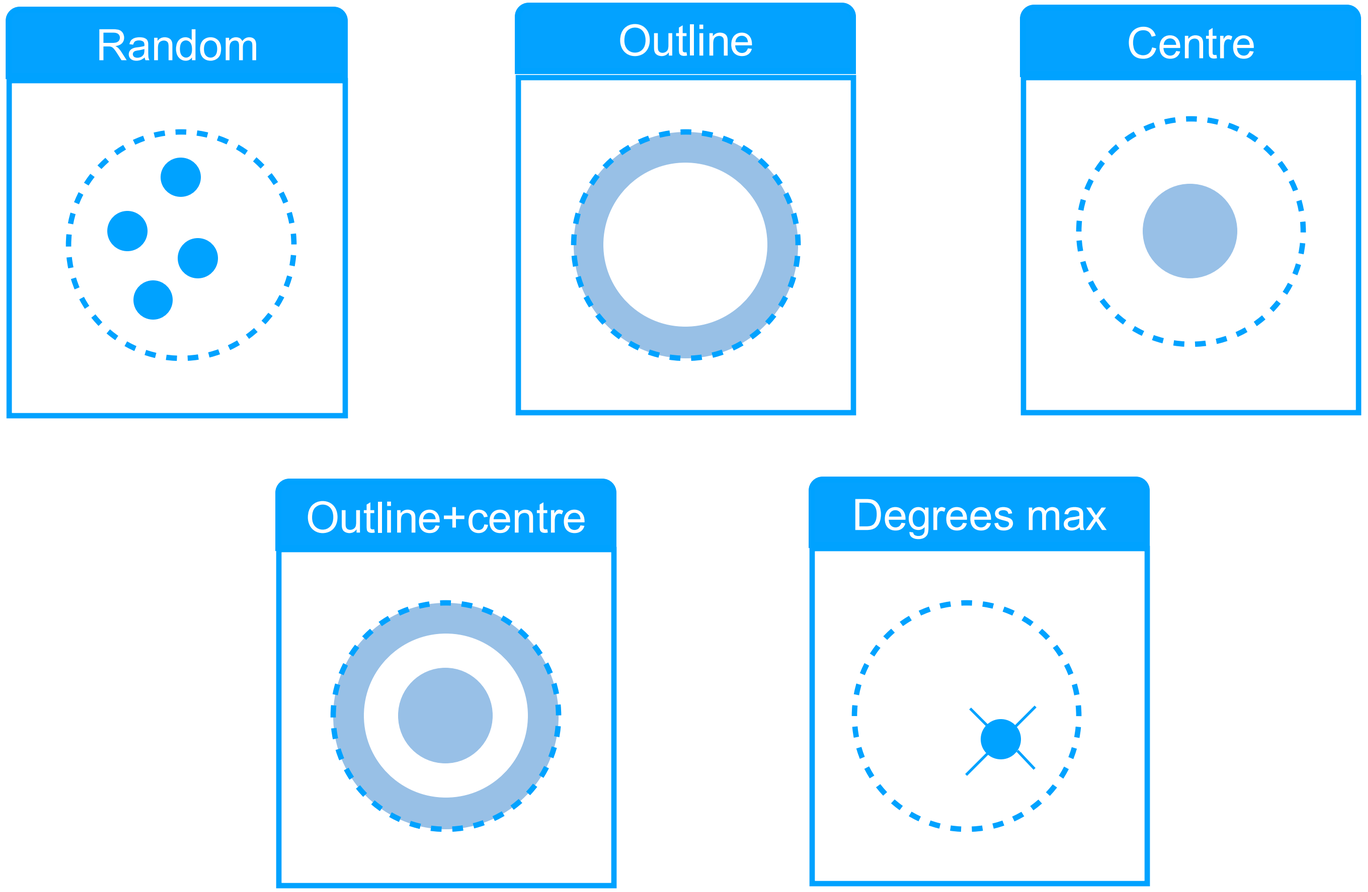



**P** = A
$dist_{max}(*, \mathbf{P}) + dist_{max}(\mathbf{P}, *) = 3 + 6 = 9$
$H - optCost - cost_{max} = 15 - 1 - 4 = 10$

$dist_{max}(*, \mathbf{P}) + dist_{max}(\mathbf{P}, *) \leq H - optCost - cost_{max}$

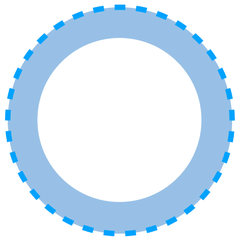
# Our approach

How to select the landmark?



# Our approach

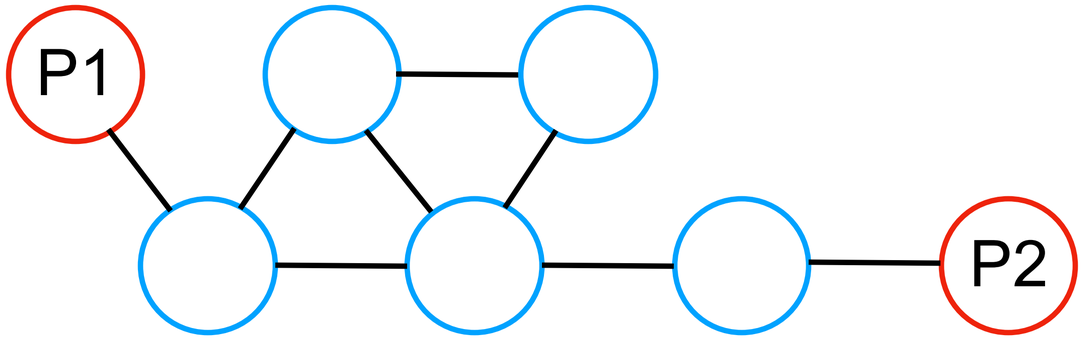
Outline



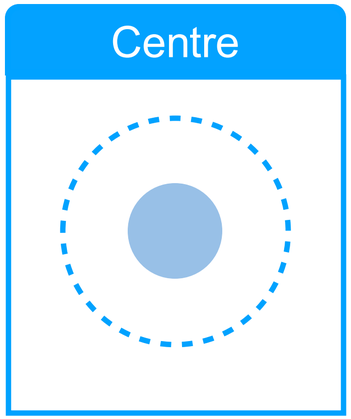
Graph outline

The outline of a graph G is one or more pairs of nodes (a, b) that maximise the shortest path between a and b.

- A node is selected
- P1: the most distant node
- P2: the most distant node from P1



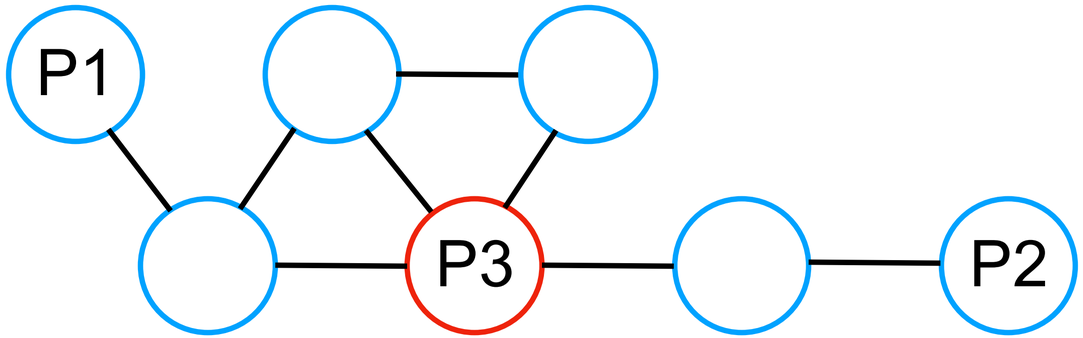
# Our approach



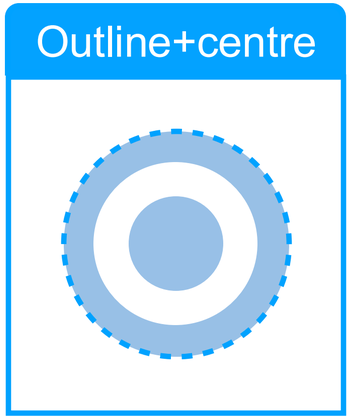

**Graph centre**

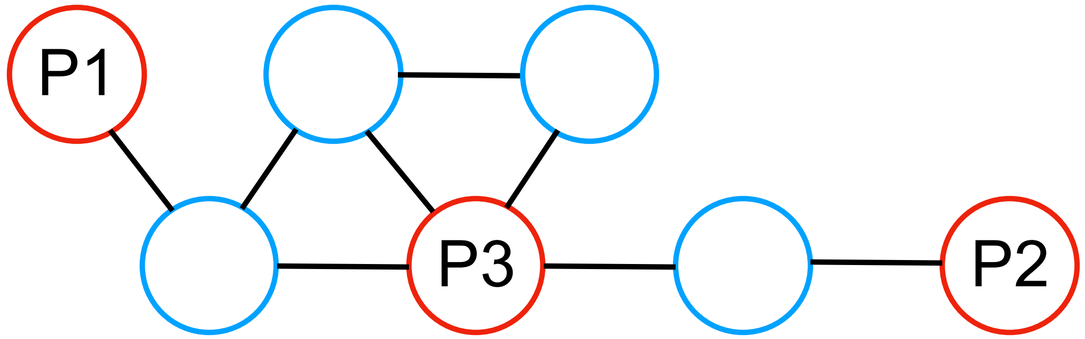
The centre of a graph G is one or more nodes which each minimize the maximum distance to all the other nodes of G.

- Select a node
- P1: the most distant node
- P2: the most distant node from P1
- P3: the node halfway between P1 and P2

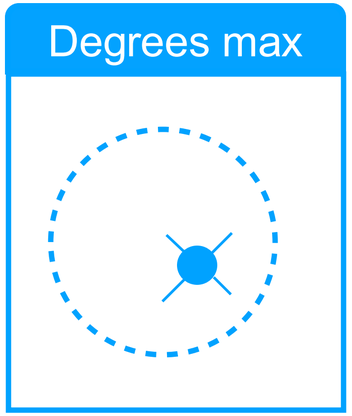


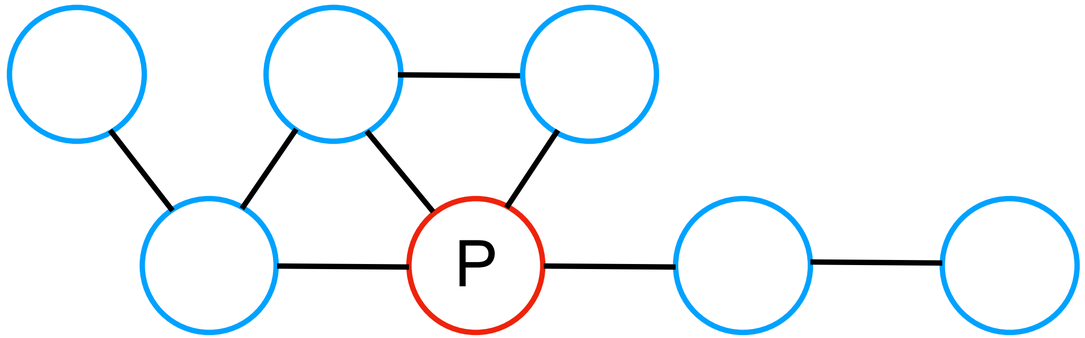
# Our approach



# Our approach



# Results

# Results

## Selected data

- Travelling salesman (TSP)
  - Real data
- Stocking cost
  - Highly structured random data
- Assign tasks to machines (FJSSP)
  - Data from several papers
- Assign children to activities (CHILD)
  - Real data

# Results

## Selected data

- Computation of H:
  - TSP:  Heuristic of Lin-Kernighan
  - Regular H: the smallest value such that there is a solution
  - Big H: Regular H × 2

# Results

- Evolution of the number of shortest paths

We are equivalent or we compute fewer shorter paths

| | Régin | Landmark Number | C | O | C & O | Deg | R |
|---|---|---|---|---|---|---|---|
| TSP ($\leq$ 100 cities) | 57.6 | 1 | 31.7 | 36.3 | 36.2 | 27.7 | 27.7 |
| | | 2 | 35.3 | 39.9 | 39.8 | 32.5 | 29.5 |
| | | 3 | 38 | 42.7 | 42.5 | 32.5 | 28.5 |
| | | 4 | 41.6 | 46.3 | 46.1 | 32 | 30.1 |
| | | 5+ | 44.8 | 50 | 50.2 | 32 | 32.2 |
| TSP ($>$ 100 & $<$ 250 cities) | 163.3 | 1 | 42.2 | 45 | 47.9 | 40.5 | 40.5 |
| | | 2 | 44.4 | 47.4 | 46.3 | 41.6 | 41.6 |
| | | 3 | 46 | 49.3 | 48.2 | 41.2 | 41.2 |
| | | 4 | 48.6 | 51.9 | 50.8 | 42.3 | 42.3 |
| | | 5+ | 50.2 | 54.1 | 52.2 | 43.1 | 43.3 |
| TSP ($\geq$ 250 cities) | 662.7 | 1 | 18.1 | 19.8 | 19.8 | 17.8 | 17.8 |
| | | 2 | 18.5 | 21.4 | 21.4 | 18.1 | 18.1 |
| | | 3 | 18.5 | 21 | 19.3 | 16.3 | 16.2 |
| | | 4 | 18.8 | 21.6 | 19.9 | 16.4 | 16.3 |
| | | 5+ | 19 | 21.8 | 20.1 | 16.7 | 16.4 |

# Results

- Evolution of the number of shortest paths

We are equivalent or we compute fewer shorter paths

With Big H we compute much fewer shortest paths

| | Régin | Landmark Number | C | O | C & O | Deg | R |
|---|---|---|---|---|---|---|---|
| StockingCost (Regular H) | 493.3 | 1 | 496.9 | 497.3 | 496.9 | 495.3 | 495.3 |
| | | 2 | 500.8 | 501.2 | 500.8 | 497.3 | 497.2 |
| | | 3 | 504.7 | 505.1 | 504.7 | 499.2 | 499.1 |
| | | 4 | 508.6 | 509 | 508.6 | 501.2 | 501 |
| | | 5+ | 512.5 | 512.9 | 512.6 | 503.2 | 503 |
| StockingCost (Big H) | 493.3 | 1 | 4 | 4 | 4 | 2 | 2 |
| | | 2 | 4 | 4 | 4 | 2 | 2 |
| | | 3 | 4 | 4 | 4 | 2 | 2 |
| | | 4 | 4 | 4 | 4 | 2 | 2 |
| | | 5+ | 4 | 4 | 4 | 2 | 2 |

# Results

- Evolution of the number of shortest paths

We are equivalent or we compute fewer shorter paths

With Big H we compute much fewer shortest paths

From 4 landmarks the number of shortest paths no longer decreases

| | Régin | Landmark Number | C | O | C & O | Deg | R |
|---|---|---|---|---|---|---|---|
| FJSSP (Regular H) | 10.4 | 1 | 8.3 | 5.1 | 4.8 | 2 | 6.3 |
| | | 2 | 8.3 | 5.1 | 4.8 | 2 | 5.3 |
| | | 3 | 8.3 | 5.1 | 4.8 | 2 | 4.6 |
| | | 4 | 8.3 | 5.1 | 4.8 | 2 | 4 |
| | | 5+ | 8.3 | 5.1 | 4.8 | 2 | 4 |
| FJSSP (Big H) | 10.4 | 1 | 4.5 | 4.3 | 4.3 | 2 | 3.2 |
| | | 2 | 4.5 | 4.3 | 4.3 | 2 | 2.8 |
| | | 3 | 4.5 | 4.3 | 4.3 | 2 | 2.6 |
| | | 4 | 2.9 | 4.3 | 4.3 | 2 | 2.4 |
| | | 5+ | 2.9 | 4.3 | 4.3 | 2 | 2.4 |

# Results

- Evolution of resolution time (ms) with 4 landmarks

We are equivalent or faster

| | | Régin | C | O | C & O | Deg | R |
|---|---|---|---|---|---|---|---|
| TSP ($\leq$ 100 cities) | Mean | 7.3 | 5.9 | 6 | 6.6 | 5.7 | **4.5** |
| | Median | 3.4 | 3.6 | 4.4 | 4.1 | 3.6 | **3.3** |
| | Ratio | | 1.2 | 1.2 | 1.1 | 1.3 | **1.6** |
| TSP ($>$ 100 & $<$ 250 cities) | Mean | 76.6 | **29.8** | 30.6 | 30.2 | **28.6** | 31.1 |
| | Median | 51.2 | **14.3** | 16 | 17 | 15.4 | **14.3** |
| | Ratio | | **2.6** | 2.5 | 2.5 | **2.7** | 2.5 |
| TSP ($\geq$ 250 cities) | Mean | 12124.9 | 278.9 | 275.2 | 275.4 | **213** | 265 |
| | Median | 2310.2 | 126.8 | 117.7 | 90.6 | 89.1 | **85.9** |
| | Ratio | | 43.5 | 44.1 | 44 | **56.9** | 45.8 |
| StockingCost (Regular H) | Mean | 603.83 | **511.8** | 617.9 | 626.2 | 580.3 | 639.4 |
| | Median | 585.7 | 553.3 | 186.9 | 186.4 | 248 | **166.4** |
| | Ratio | | **1.2** | 1 | 1 | 1 | 0.9 |
| StockingCost (Big H) | Mean | 534.76 | 34.1 | 32.4 | **31.6** | 33.2 | 32.6 |
| | Median | 519.1 | 33.8 | 32.4 | 31.9 | 32.8 | **30.1** |
| | Ratio | | 15.7 | 16.5 | **16.9** | 16 | 16.4 |
| FJSSP (Regular H) | Mean | 0.4 | 0.5 | **0.3** | 0.4 | 0.4 | 0.5 |
| | Median | **0.1** | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 |
| | Ratio | | 0.8 | **1.7** | 0.75 | 1 | 0.8 |
| FJSSP (Big H) | Mean | 0.4 | 0.4 | **0.3** | **0.3** | **0.3** | **0.3** |
| | Median | **0.1** | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| | Ratio | | 1 | **1.3** | **1.3** | **1.3** | **1.3** |
| CHILD (Regular H) | Time | 65.1 | 69.2 | **54.4** | 67.6 | 75.9 | 65.4 |
| | Ratio | | 0.9 | **1.2** | 1 | 0.8 | 1 |
| CHILD (Big H) | Time | 58.2 | 7 | 6.5 | 7.3 | **6** | **6** |
| | Ratio | | 8.3 | 9 | 8 | **9.7** | **9.7** |

# Results

- Evolution of resolution time (ms) with 4 landmarks

We are equivalent or faster

With Big H we are much faster

| | | Régin | C | O | C & O | Deg | R |
|---|---|---|---|---|---|---|---|
| TSP ($\leq$ 100 cities) | Mean | 7.3 | 5.9 | 6 | 6.6 | 5.7 | **4.5** |
| | Median | 3.4 | 3.6 | 4.4 | 4.1 | 3.6 | **3.3** |
| | Ratio | | 1.2 | 1.2 | 1.1 | 1.3 | **1.6** |
| TSP ($>$ 100 & $<$ 250 cities) | Mean | 76.6 | **29.8** | 30.6 | 30.2 | **28.6** | 31.1 |
| | Median | 51.2 | **14.3** | 16 | 17 | 15.4 | **14.3** |
| | Ratio | | **2.6** | 2.5 | 2.5 | **2.7** | 2.5 |
| TSP ($\geq$ 250 cities) | Mean | 12124.9 | 278.9 | 275.2 | 275.4 | **213** | 265 |
| | Median | 2310.2 | 126.8 | 117.7 | 90.6 | 89.1 | **85.9** |
| | Ratio | | 43.5 | 44.1 | 44 | **56.9** | 45.8 |
| StockingCost (Regular H) | Mean | 603.83 | **511.8** | 617.9 | 626.2 | 580.3 | 639.4 |
| | Median | 585.7 | 553.3 | 186.9 | 186.4 | 248 | **166.4** |
| | Ratio | | **1.2** | 1 | 1 | 1 | 0.9 |
| StockingCost (Big H) | Mean | 534.76 | 34.1 | 32.4 | **31.6** | 33.2 | 32.6 |
| | Median | 519.1 | 33.8 | 32.4 | 31.9 | 32.8 | **30.1** |
| | Ratio | | 15.7 | 16.5 | **16.9** | 16 | 16.4 |
| FJSSP (Regular H) | Mean | 0.4 | 0.5 | **0.3** | 0.4 | 0.4 | 0.5 |
| | Median | **0.1** | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 |
| | Ratio | | 0.8 | **1.7** | 0.75 | 1 | 0.8 |
| FJSSP (Big H) | Mean | 0.4 | 0.4 | **0.3** | **0.3** | **0.3** | **0.3** |
| | Median | **0.1** | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| | Ratio | | 1 | **1.3** | **1.3** | **1.3** | **1.3** |
| CHILD (Regular H) | Time | 65.1 | 69.2 | **54.4** | 67.6 | 75.9 | 65.4 |
| | Ratio | | 0.9 | **1.2** | 1 | 0.8 | 1 |
| CHILD (Big H) | Time | 58.2 | 7 | 6.5 | 7.3 | **6** | **6** |
| | Ratio | | 8.3 | 9 | 8 | **9.7** | **9.7** |

# Results

- Evolution of resolution time (ms) with 4 landmarks

We are equivalent or faster

With Big H we are much faster

Selection by degrees and random are slightly better

| | | Régin | C | O | C & O | Deg | R |
|---|---|---|---|---|---|---|---|
| TSP ($\leq$ 100 cities) | Mean | 7.3 | 5.9 | 6 | 6.6 | 5.7 | **4.5** |
| | Median | 3.4 | 3.6 | 4.4 | 4.1 | 3.6 | **3.3** |
| | Ratio | | 1.2 | 1.2 | 1.1 | 1.3 | **1.6** |
| TSP ($>$ 100 & $<$ 250 cities) | Mean | 76.6 | **29.8** | 30.6 | 30.2 | **28.6** | 31.1 |
| | Median | 51.2 | **14.3** | 16 | 17 | 15.4 | **14.3** |
| | Ratio | | **2.6** | 2.5 | 2.5 | **2.7** | 2.5 |
| TSP ($\geq$ 250 cities) | Mean | 12124.9 | 278.9 | 275.2 | 275.4 | **213** | 265 |
| | Median | 2310.2 | 126.8 | 117.7 | 90.6 | 89.1 | **85.9** |
| | Ratio | | 43.5 | 44.1 | 44 | **56.9** | 45.8 |
| StockingCost (Regular H) | Mean | 603.83 | **511.8** | 617.9 | 626.2 | 580.3 | 639.4 |
| | Median | 585.7 | 553.3 | 186.9 | 186.4 | 248 | **166.4** |
| | Ratio | | **1.2** | 1 | 1 | 1 | 0.9 |
| StockingCost (Big H) | Mean | 534.76 | 34.1 | 32.4 | **31.6** | 33.2 | 32.6 |
| | Median | 519.1 | 33.8 | 32.4 | 31.9 | 32.8 | **30.1** |
| | Ratio | | 15.7 | 16.5 | **16.9** | 16 | 16.4 |
| FJSSP (Regular H) | Mean | 0.4 | 0.5 | **0.3** | 0.4 | 0.4 | 0.5 |
| | Median | **0.1** | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 |
| | Ratio | | 0.8 | **1.7** | 0.75 | 1 | 0.8 |
| FJSSP (Big H) | Mean | 0.4 | 0.4 | **0.3** | **0.3** | **0.3** | **0.3** |
| | Median | **0.1** | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| | Ratio | | 1 | **1.3** | **1.3** | **1.3** | **1.3** |
| CHILD (Regular H) | Time | 65.1 | 69.2 | **54.4** | 67.6 | 75.9 | 65.4 |
| | Ratio | | 0.9 | **1.2** | 1 | 0.8 | 1 |
| CHILD (Big H) | Time | 58.2 | 7 | 6.5 | 7.3 | **6** | **6** |
| | Ratio | | 8.3 | 9 | 8 | **9.7** | **9.7** |

# Results

- Evolution of resolution time (ms) with 4 landmarks

We are equivalent or faster

With Big H we are much faster

Selection by degrees and random are slightly better

| | | Régin | C | O | C & O | Deg | R |
|---|---|---|---|---|---|---|---|
| TSP ($\leq$ 100 cities) | Mean | 7.3 | 5.9 | 6 | 6.6 | 5.7 | **4.5** |
| | Median | 3.4 | 3.6 | 4.4 | 4.1 | 3.6 | **3.3** |
| | Ratio | | 1.2 | 1.2 | 1.1 | 1.3 | **1.6** |
| TSP ($>$ 100 & $<$ 250 cities) | Mean | 76.6 | **29.8** | 30.6 | 30.2 | **28.6** | 31.1 |
| | Median | 51.2 | **14.3** | 16 | 17 | 15.4 | **14.3** |
| | Ratio | | **2.6** | 2.5 | 2.5 | **2.7** | 2.5 |
| TSP ($\geq$ 250 cities) | Mean | 12124.9 | 278.9 | 275.2 | 275.4 | **213** | 265 |
| | Median | 2310.2 | 126.8 | 117.7 | 90.6 | 89.1 | **85.9** |
| | Ratio | | 43.5 | 44.1 | 44 | **56.9** | 45.8 |
| StockingCost (Regular H) | Mean | 603.83 | **511.8** | 617.9 | 626.2 | 580.3 | 639.4 |
| | Median | 585.7 | 553.3 | 186.9 | 186.4 | 248 | **166.4** |
| | Ratio | | **1.2** | 1 | 1 | 1 | 0.9 |
| StockingCost (Big H) | Mean | 534.76 | 34.1 | 32.4 | **31.6** | 33.2 | 32.6 |
| | Median | 519.1 | 33.8 | 32.4 | 31.9 | 32.8 | **30.1** |
| | Ratio | | 15.7 | 16.5 | **16.9** | 16 | 16.4 |
| FJSSP (Regular H) | Mean | 0.4 | 0.5 | **0.3** | 0.4 | 0.4 | 0.5 |
| | Median | **0.1** | 0.3 | 0.2 | 0.3 | 0.2 | 0.3 |
| | Ratio | | 0.8 | **1.7** | 0.75 | 1 | 0.8 |
| FJSSP (Big H) | Mean | 0.4 | 0.4 | **0.3** | **0.3** | **0.3** | **0.3** |
| | Median | **0.1** | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| | Ratio | | 1 | **1.3** | **1.3** | **1.3** | **1.3** |
| CHILD (Regular H) | Time | 65.1 | 69.2 | **54.4** | 67.6 | 75.9 | 65.4 |
| | Ratio | | 0.9 | **1.2** | 1 | 0.8 | 1 |
| CHILD (Big H) | Time | 58.2 | 7 | 6.5 | 7.3 | **6** | **6** |
| | Ratio | | 8.3 | 9 | 8 | **9.7** | **9.7** |

# Conclusion

We have proposed a new improvement to detect when assignments can be part of a solution

Using landmarks is always better

In practice, it is often case that all assignments are valid

Up to 50 times faster

# Thank you for your attention!

Paper

Personal website