

Inverting Step-reduced SHA-1 and MD5 by Parameterized SAT Solvers

Oleg Zaikin

ISDCT SB RAS, Russia

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- 1 Cryptographic hash functions MD5 and SHA-1
- 2 Intermediate inverse problems for MD5 and SHA-1
- 3 SAT encoding
- 4 Solving intermediate inverse problems by Kissat
- 5 Tuning Kissat
- 6 Inverting 24-step SHA-1 and 29-step MD5 by parameterized Kissat

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The first two properties are obligatory.

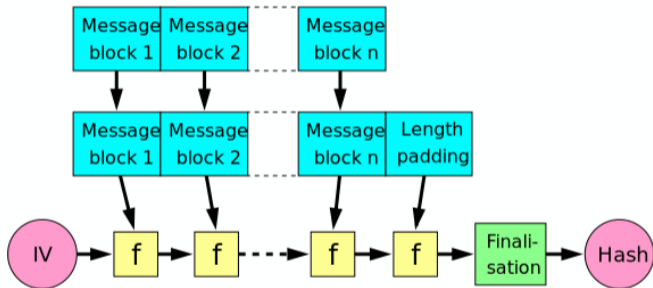
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- **Proof-of-work:** a mining reward is unlocked after some partial hash inversions (e.g. in Bitcoin).

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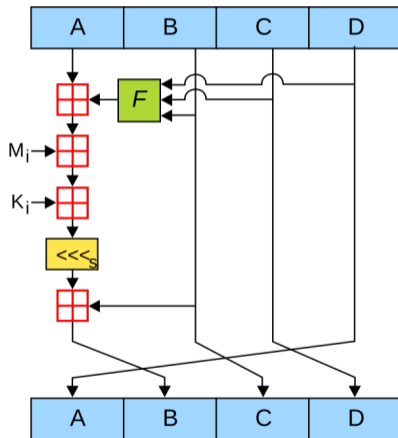


- Initialization vector (IV) has a fixed value.
- The compression function f takes the result so far, combines it with a message block, and produces an intermediate result.

- MD5 – a Merkle-Damgard-based cryptographic hash function proposed in 1992.

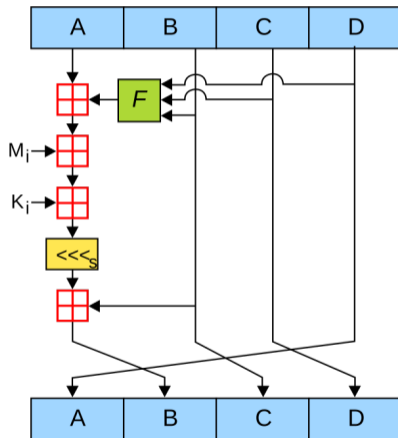
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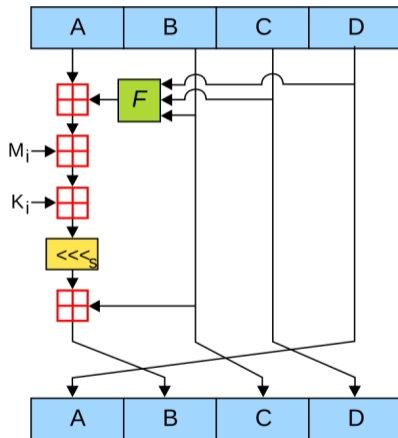
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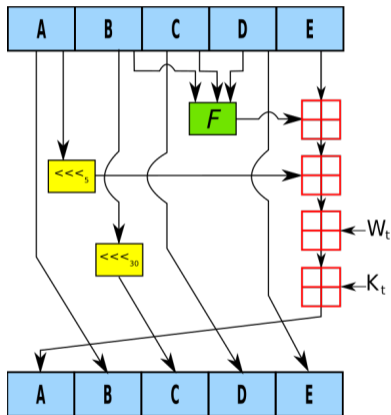
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- 64 steps; each step all registers are updated.
- Before the 1st step A, B, C, D are IV, on the last step they are hash.



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- SHA-1 – a Merkle-Damgard-based cryptographic hash function proposed in 1995.

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- Compared to MD5:
 - 1 160-bit hash.
 - 2 5 registers A, B, C, D, E .
 - 3 80 steps; 4 rounds, 20 steps each.



One SHA-1 step². F is a round function.

²<https://en.wikipedia.org/wiki/SHA-1>

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- **Goal: invert 29-step MD5 and 24-step SHA-1 via SAT.**

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The $(i + 1)$ -th step of MD5:

$$temp \leftarrow Func(B, C, D) \boxplus A \boxplus K[i] \boxplus M[g]$$
$$A \leftarrow D$$
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Idea: assign constant values to several bits in $M[g]$ in step $i + 1$ thus forming a family of intermediate inverse problems between steps i and $i + 1$.

Intermediate inverse problems for MD5

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A weakened $(i + 1)$ -th step of MD5:

$$weakM \leftarrow (M[g] \ggg (32 - j)) \lll (32 - j)$$

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- The j -th intermediate MD5 function between i and $i + 1$, is called $(i \ j/32)$ -step MD5.

Intermediate inverse problems for SHA-1

- The main hardness lies in $M[g]$ as well.
- 31 intermediate inverse problems are formed in the same way as for MD5.

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- In the present study, the implementation is extended to maintain MD5.

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 - ② The rightmost $32 - j$ bits of weakM are assigned to 0 via adding unit clauses.
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Table: Characteristics of CNFs.

Hash	Steps	Variables	Clauses	Literals
SHA-1	23	4 288	132 672	873 727
SHA-1	23 16/32	4 480	138 812	913 700
SHA-1	24	4 448	138 764	913 620

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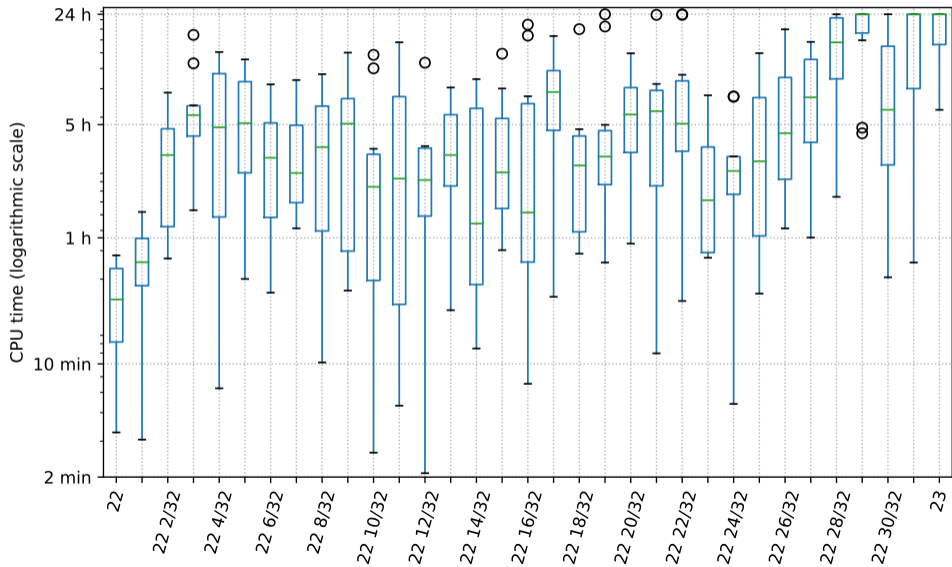
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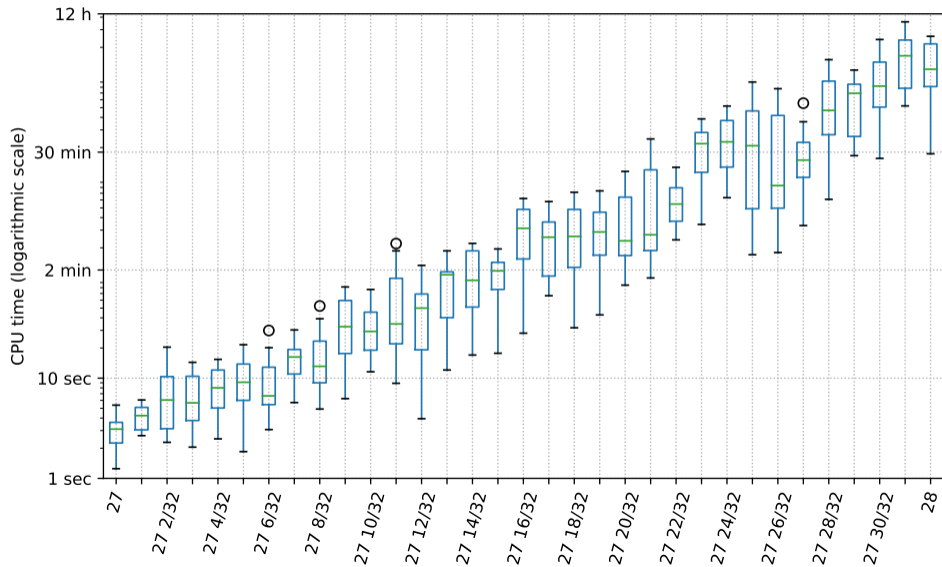
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- PC with 16-core CPU.
- Time limit 24 hours.

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Boxplots for SHA-1



Boxplots for MD5



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- **Solution:** a metaheuristic algorithm tunes the solver without checking all sets.
- **Idea: to invert $i + 1$ steps, tune Kissat on intermediate inverse problems between steps i and $i + 1$.**

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- Implementations of metaheuristic algorithms: SMAC3, PyDGGA.
- (1+1)-EA (Evolutionary Algorithm) was chosen for tuning because of its simplicity.
 - 1 Consider n parameters.
 - 2 New set of values: the value of each parameter is changed with probability $1/n$.
 - 3 Any value can be assigned, but with high probability it will be the closest to the current one.

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- The total runtime on them is **1 hour 58 minutes** on 1 CPU core.

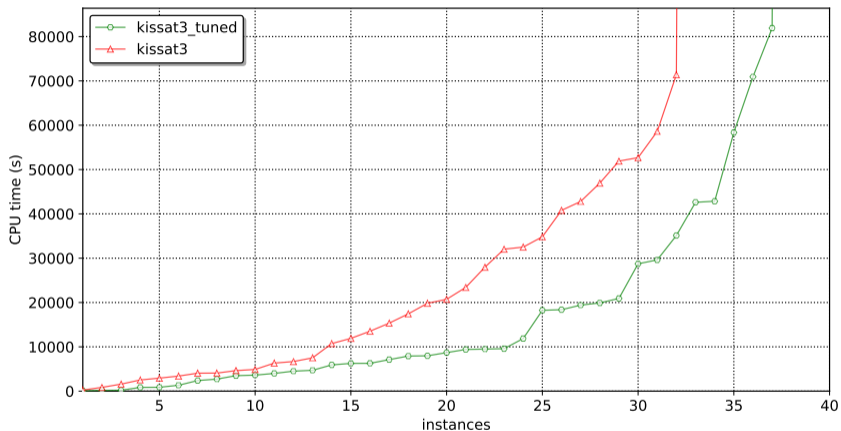
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- The total runtime on them is **1 hour 58 minutes** on 1 CPU core.
- 3 seeds for tuning, each on 16-core CPU during 24 hours.
- The best set of parameters' values: **22 minutes** in total (5 times faster).

Table: The best KISSAT's configuration found for SHA-1.

Parameter	Default value	Found value
backbonerounds	100	10
definitionticks	1 000 000	100
eliminatebound	16	32
eliminateclslim	100	10
emafast	33	10
minimizedepth	1 000	100
restartmargin	10	20
stable	1	2
sweepfliprounds	1	5
sweepmaxclauses	4 096	2 147 483 647
sweepvars	128	64
vivifytier1	3	2

Tuning Kissat for SHA-1



Comparison of the default KISSAT with its tuned version on intermediate inverse problems for steps 22-24 of SHA-1, 1-hash.

- The Cube-and-Conquer method was applied: a given formula is split via lookahead into a family of simpler subformulas, which are solved by a CDCL solver⁷.

⁷Marijn Heule et al. Cube and Conquer: Guiding CDCL SAT Solvers by Lookaheads // HVG 2011. 

- The Cube-and-Conquer method was applied: a given formula is split via lookahead into a family of simpler subformulas, which are solved by a CDCL solver⁷.
- The lookahead solver march_cu split the inverse problem for 24-step SHA-1 into 166 subformulas.

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- The Cube-and-Conquer method was applied: a given formula is split via lookahead into a family of simpler subformulas, which are solved by a CDCL solver⁷.
- The lookahead solver march_cu split the inverse problem for 24-step SHA-1 into 166 subformulas.
- The tuned Kissat was run on the subformulas on a supercomputer (166 CPU cores). A preimage was found in 23 hours.

Table: A preimage of 160 1s produced by 24-step SHA-1.

0xa6c5c463	0x182655e0	0x2c5ba5f0	0xe0028033
0x8c3779b1	0x98635880	0xc5b822e	0x297efce7
0x59987038	0xd764eca9	0x7ed9801d	0xdde4f1e0
0x524e678	0xa8ce47dc	0xa813fd76	0x8b58e09f

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- 16 CNFs in the training set: the first 16 intermediate inverse problems between steps 27-28 for 1-hash.

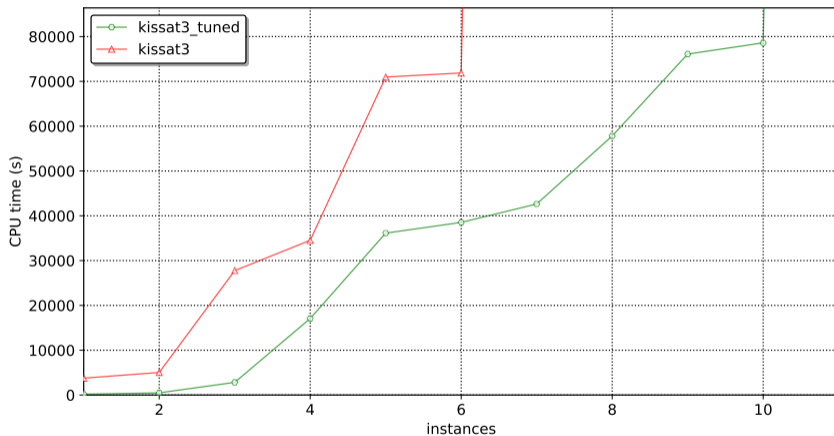
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- The best set of parameters' values: **4 minutes** in total (3 times faster).

Table: The best KISSAT's configuration found for MD5.

Parameter	Default value	Found value
chronolevels	100	1 000
decay	50	32
definitonticks	1 000 000	100
eliminatebound	16	2
eliminateocclim	2 000	1 000
emaslow	100 000	75 000
minimizedepth	1 000	100
restartmargin	10	20
shrink	3	0
stable	1	2
substituterounds	2	32
subsumeclslim	1 000	10 000
sweepmaxclauses	4 096	2 048



Comparison of the default KISSAT with its tuned version on intermediate inverse problems for steps 28-29 of MD5, 1-hash.

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- A preimage was found in 37 hours.

Table: A preimage of 128 1s produced by 29-step MD5-1.

0xe1051a9e	0x48120773	0x996a5457	0xaa1d815
0x37d8149c	0x5f999c05	0x182ba14b	0xdfff1673
0xc5db0a2f	0x44430b2a	0xa269f5a2	0x69781b85
0x2b7f0939	0xc1ff3c22	0xc55e990f	0x96ba3fb8

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Thank you for your attention! Questions?