

# Learning Effect and Compound Activities in High Multiplicity RCPSP : Application to Satellite Production

**Duc Anh Le <sup>(1)</sup> – Stéphanie Roussel <sup>(1)</sup> – Christophe Lecoutre <sup>(2)</sup> – Anouck Chan <sup>(1)</sup>**

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***(2) Artois University, CRIL***

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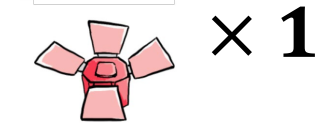
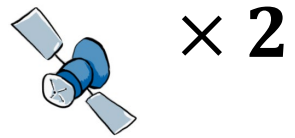
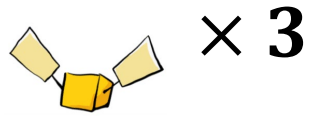
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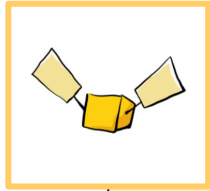
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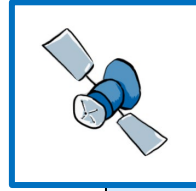
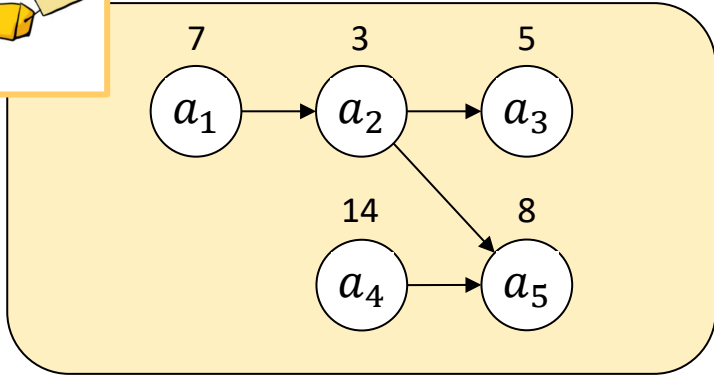
# Satellite Assembly Line



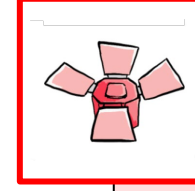
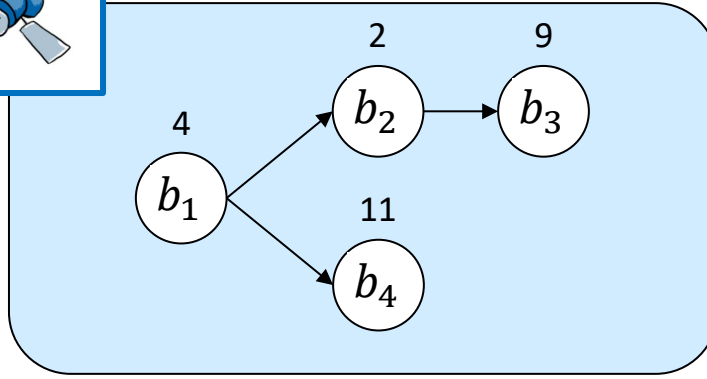
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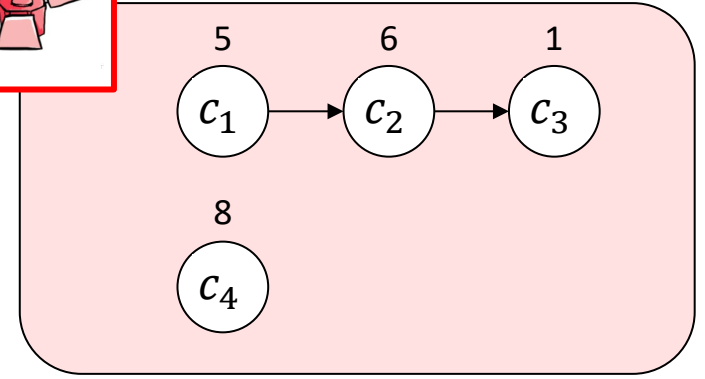
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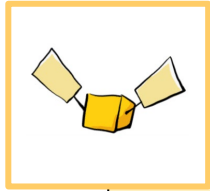
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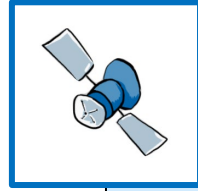
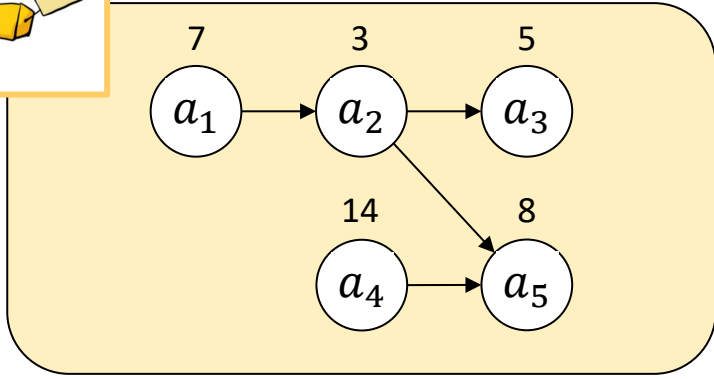
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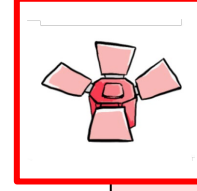
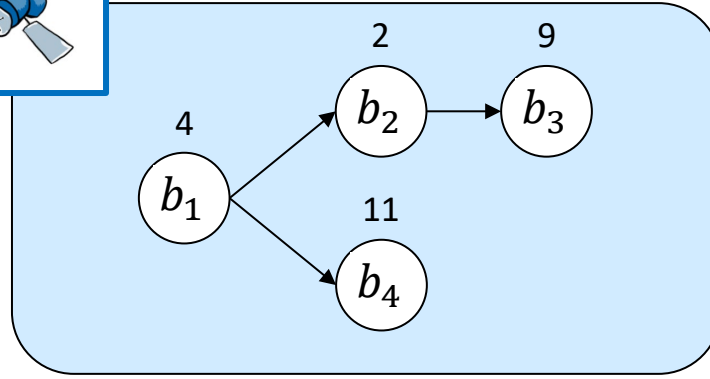
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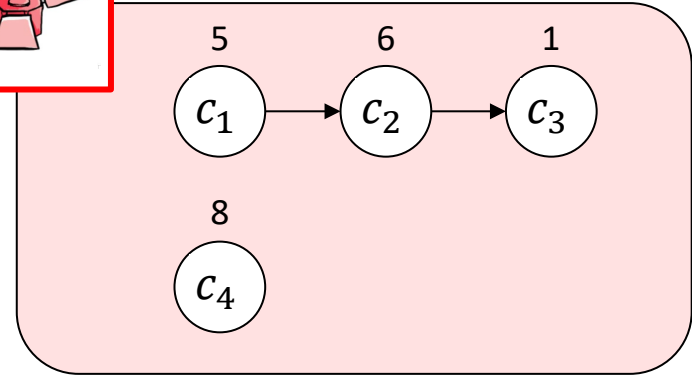
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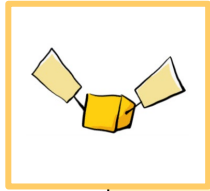
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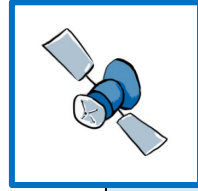
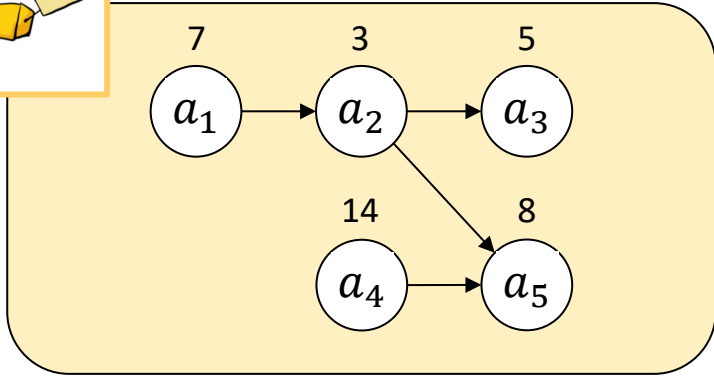
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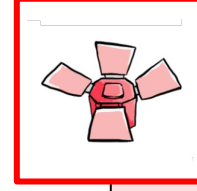
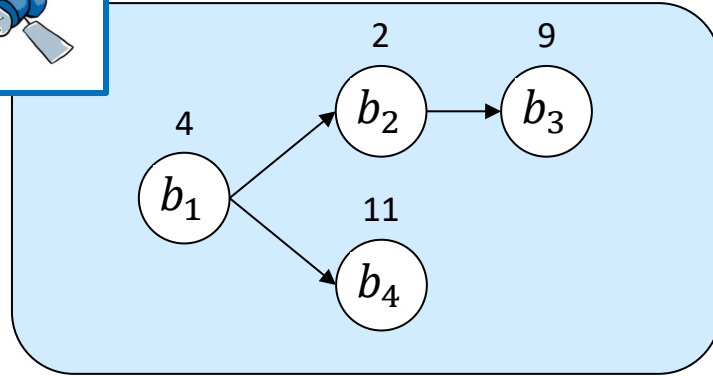
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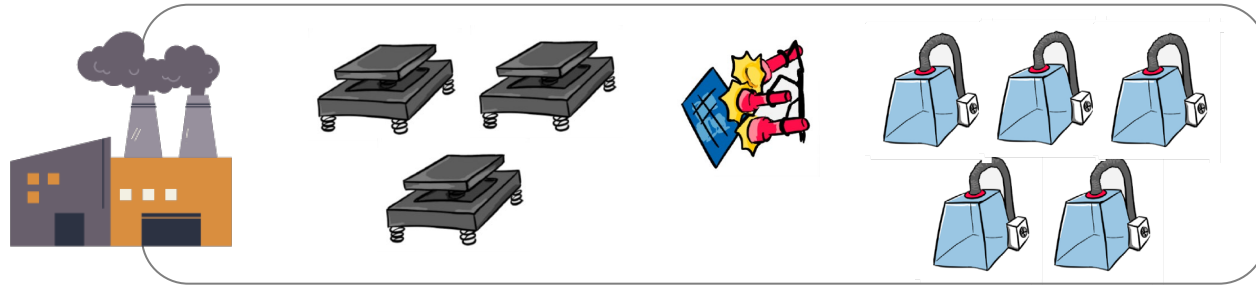
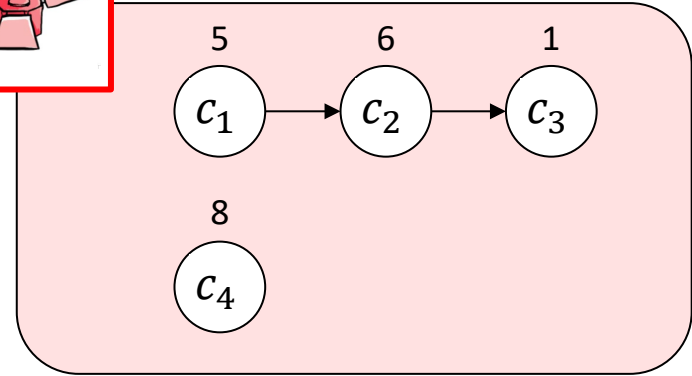
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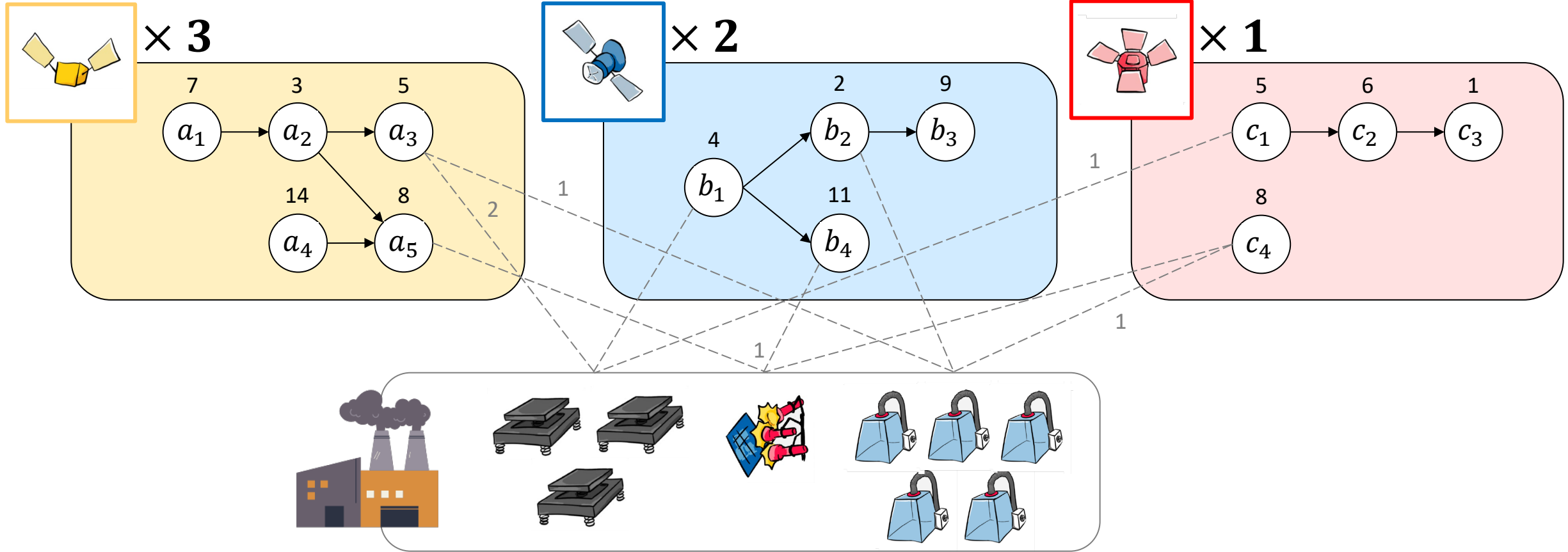
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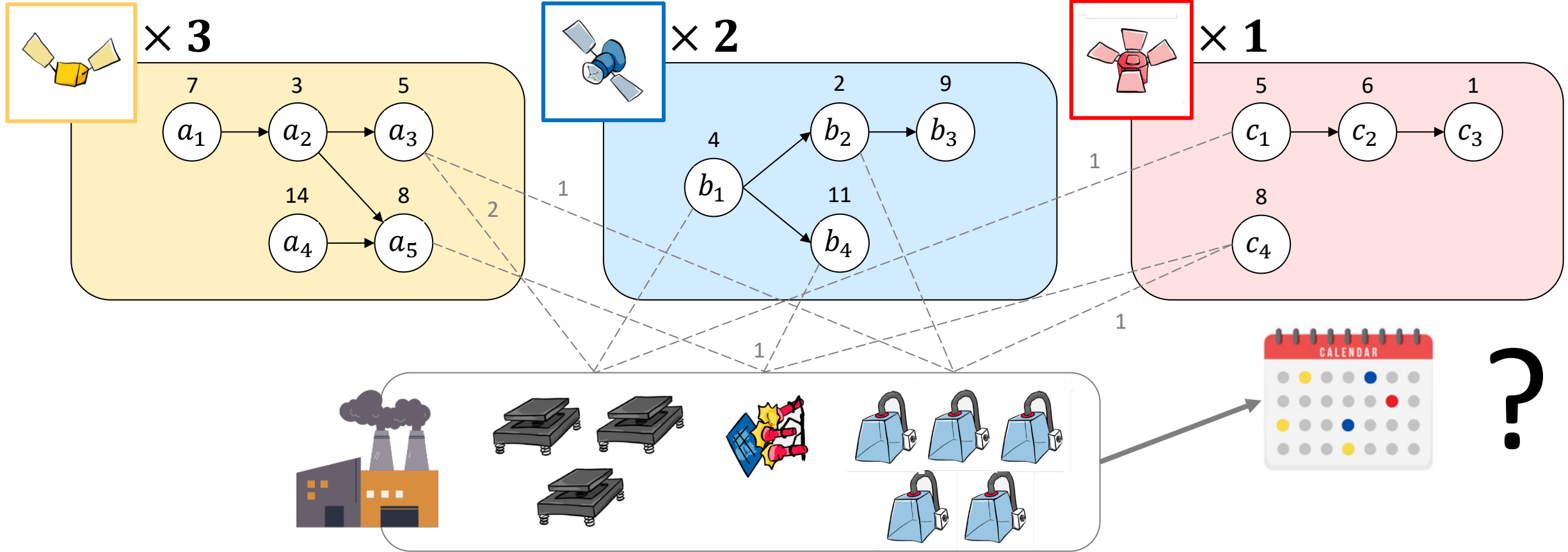
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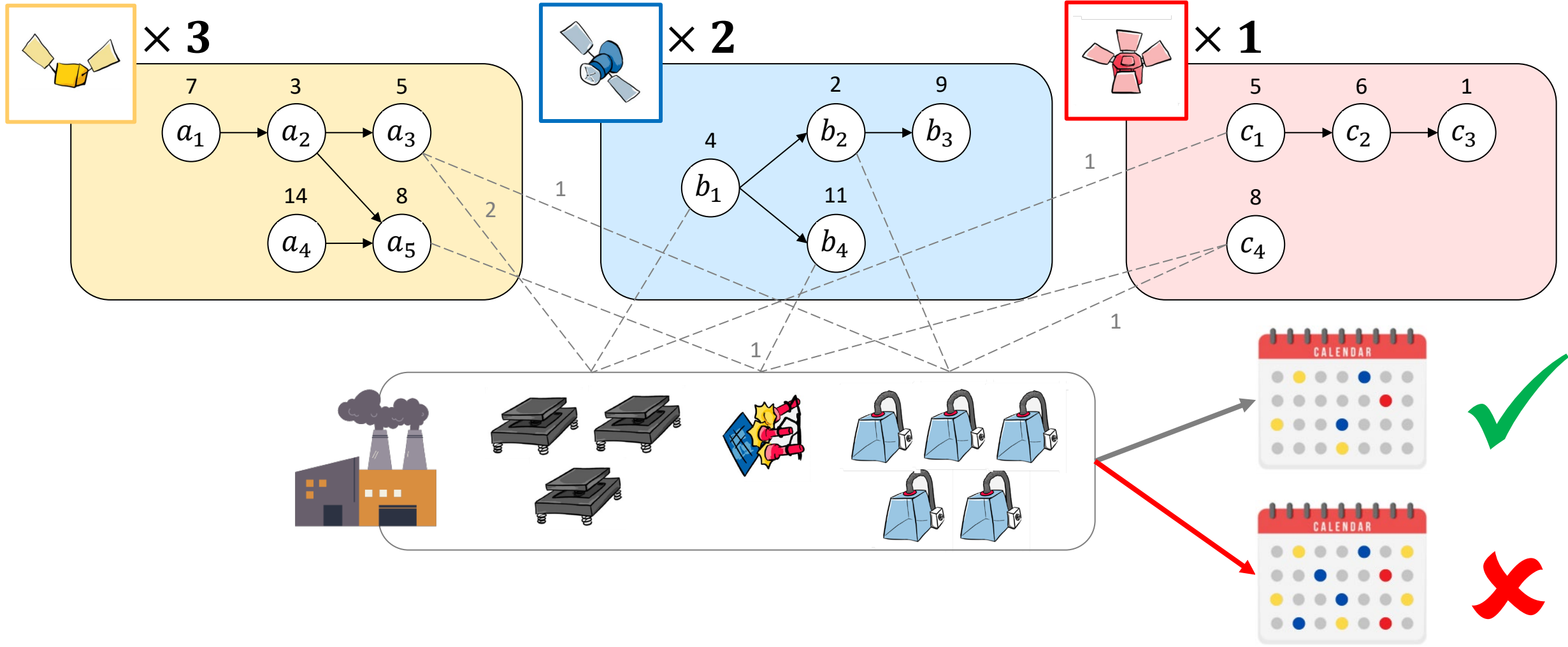


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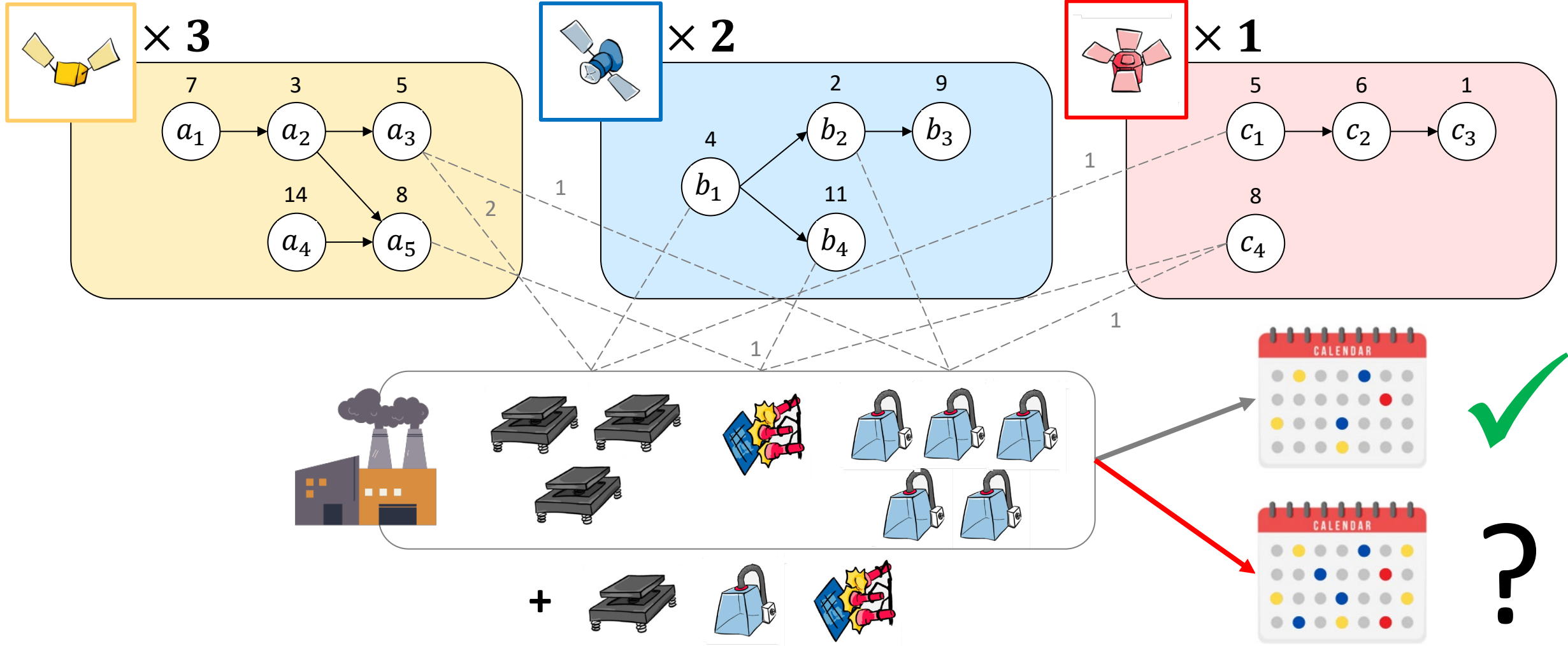




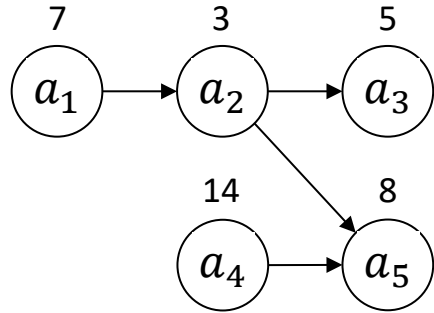
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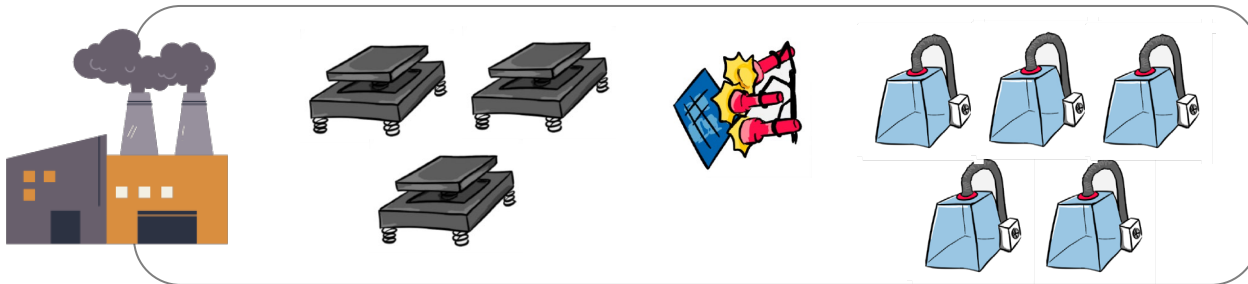
# Satellite Assembly Line



# High Multiplicity RCPSP



## Activities



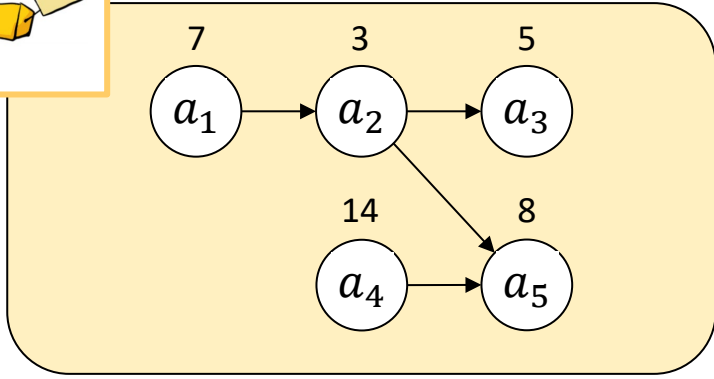
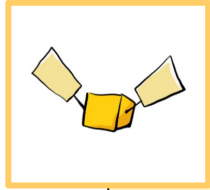
## Resources



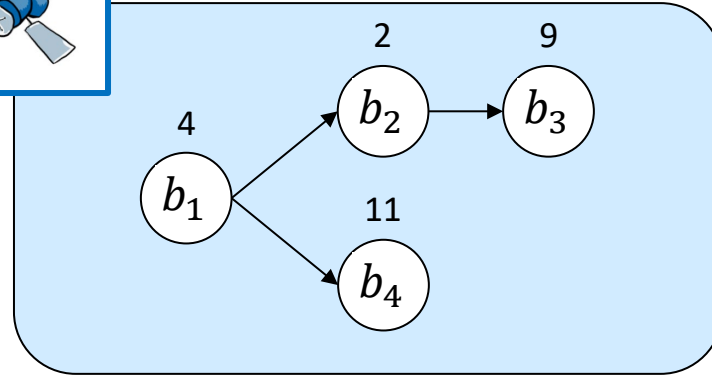
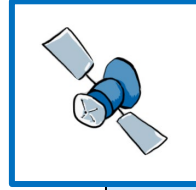
## Due date

RCPSP

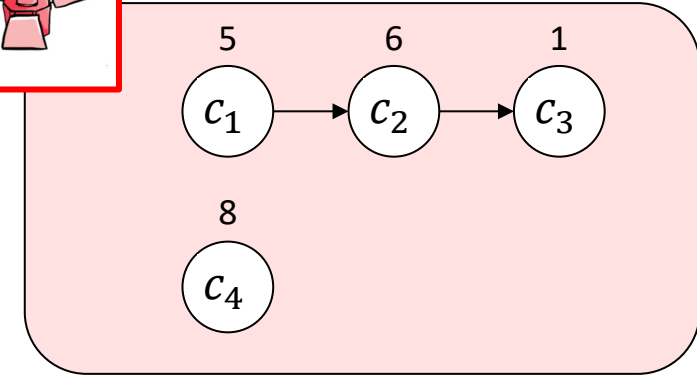
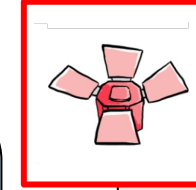
# High Multiplicity RCPSP



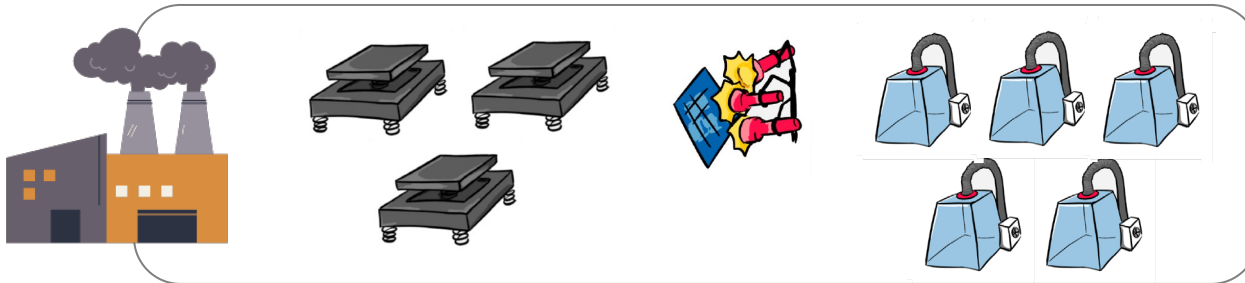
Project 1



Project 2



Project 3



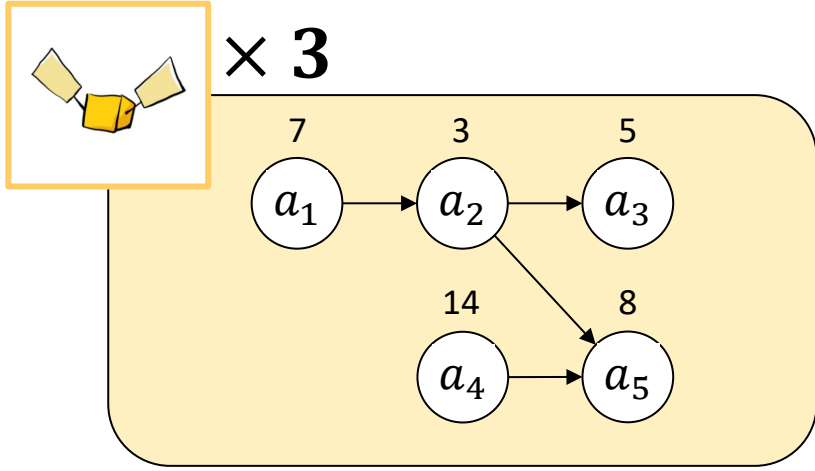
Resources



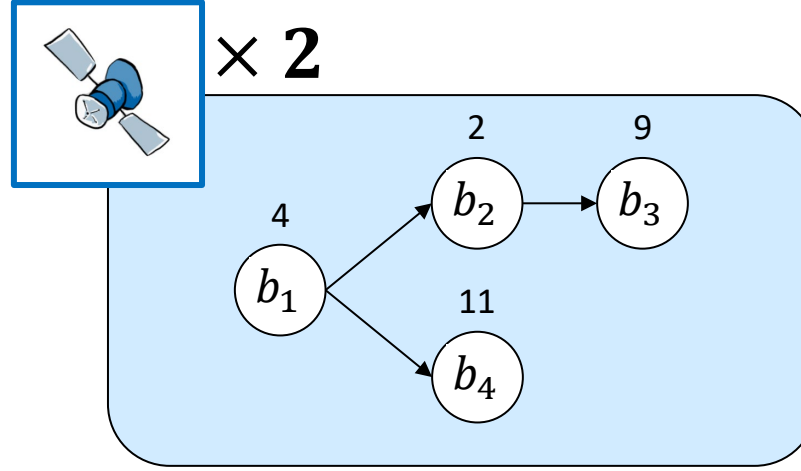
Due dates

MP-RCPSP

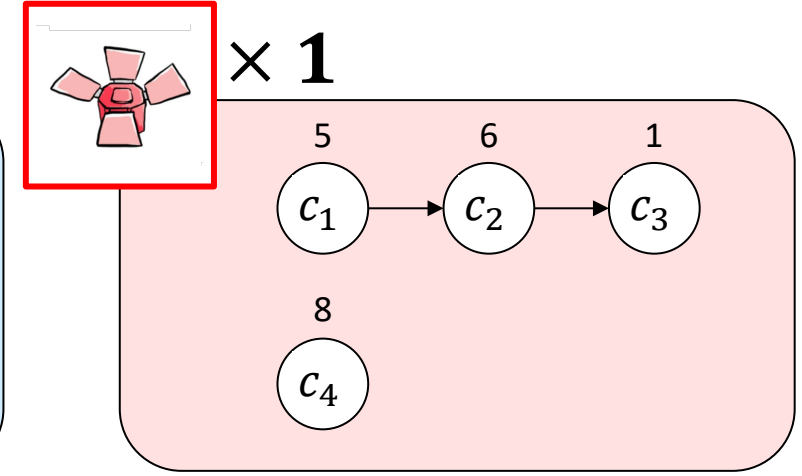
# High Multiplicity RCPSP



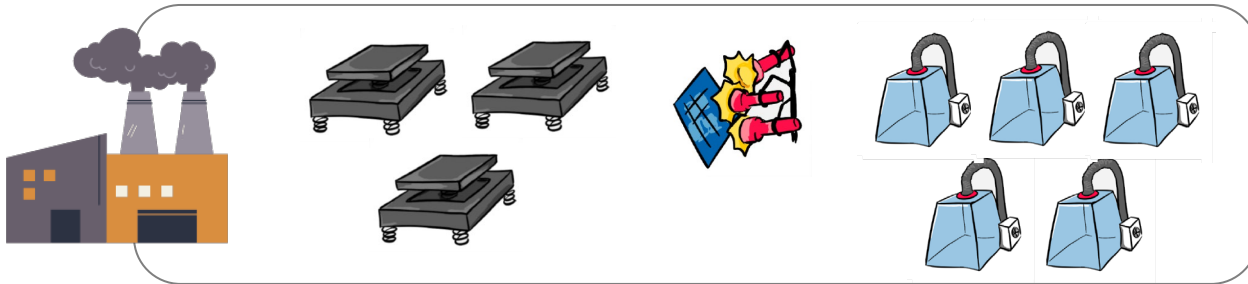
**Class 1**



**Class 2**



**Class 3**



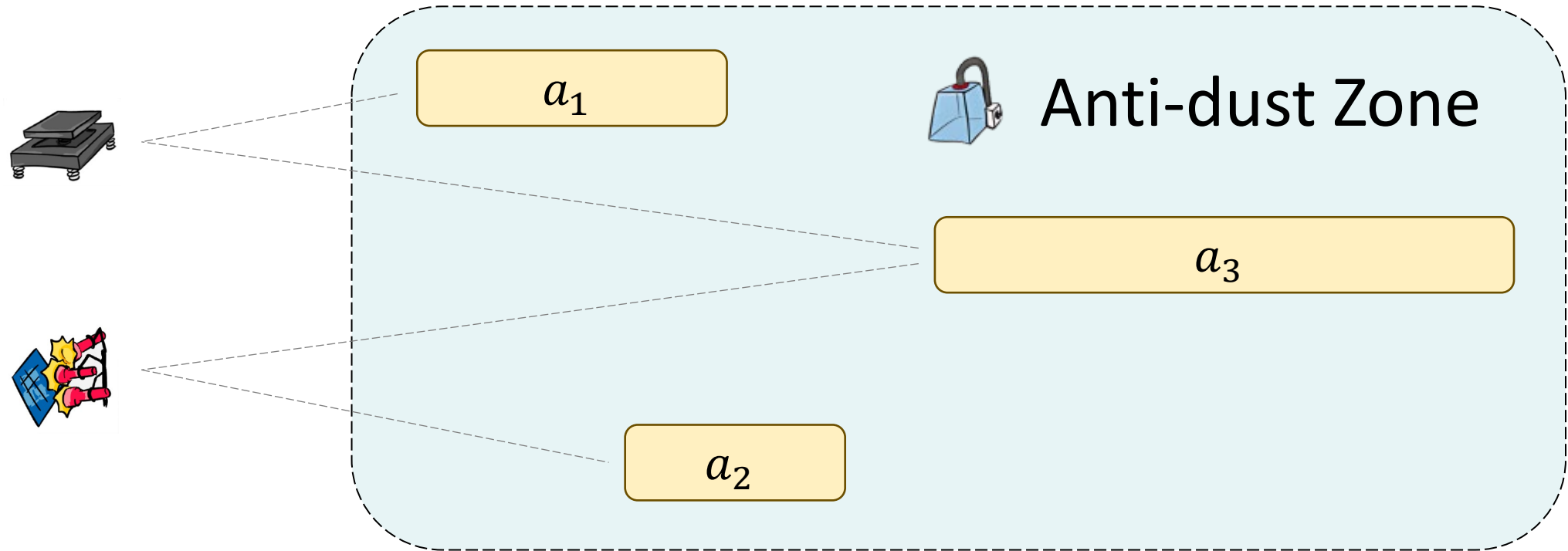
**Resources**



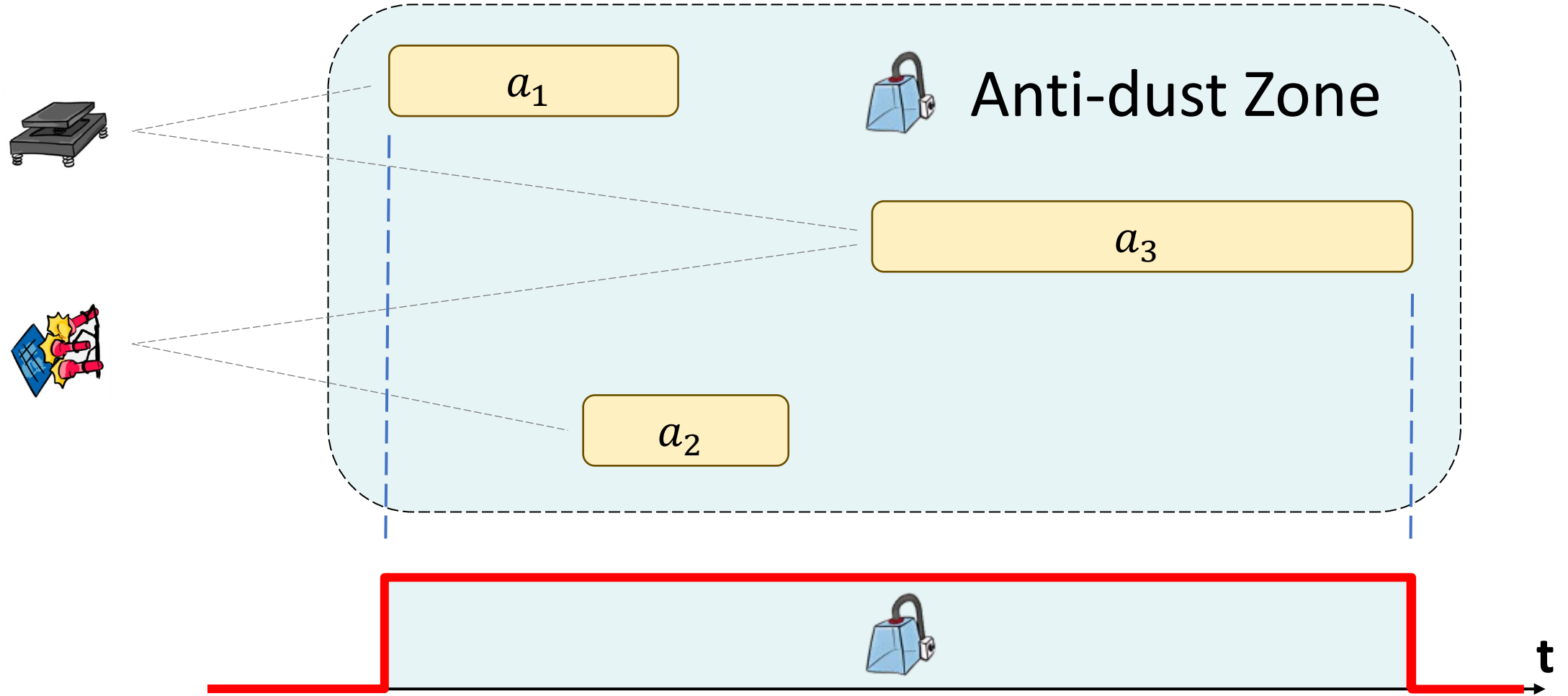
**Due dates**

**HM-RCPSP**

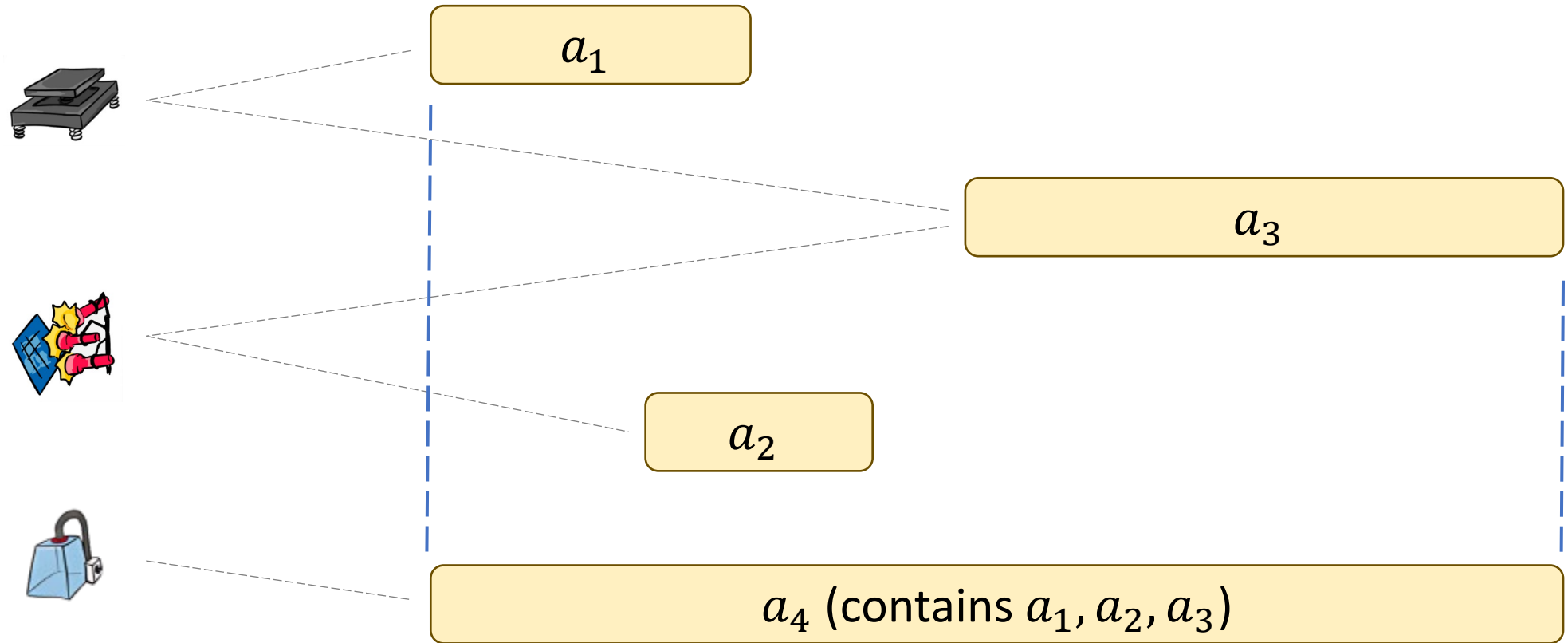
# Compound Activity



# Compound Activity



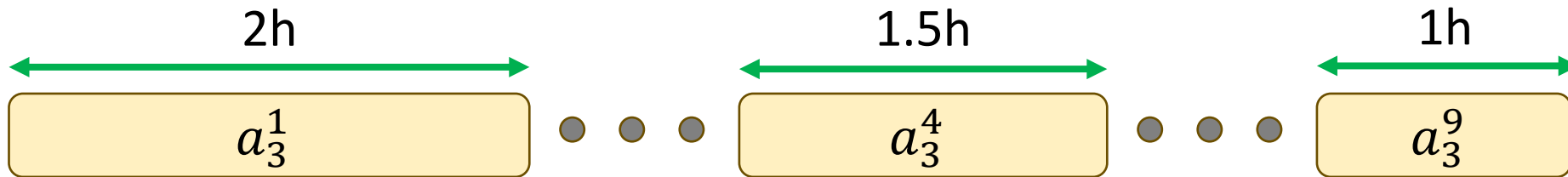
# Compound Activity





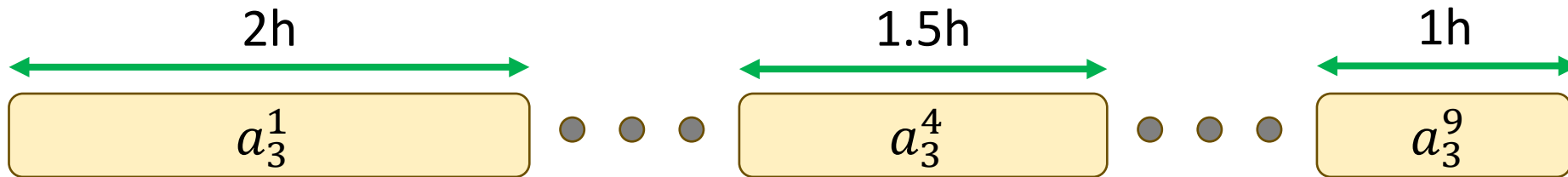
# Learning Effect

- Acquiring experience  increase in efficiency



# Learning Effect

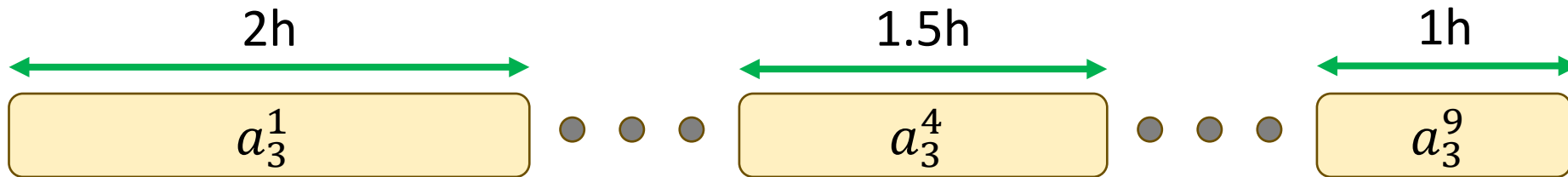
- Acquiring experience  $\rightarrow$  increase in efficiency



- The execution of activities has an impact on the duration of others

# Learning Effect

- Acquiring experience  $\rightarrow$  increase in efficiency



- The execution of activities has an impact on the duration of others

$\rightarrow$  **Projects within class are interdependent**

# State of the Art

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- RCPSP
- MP-RCPSP
- HM-RCPSP

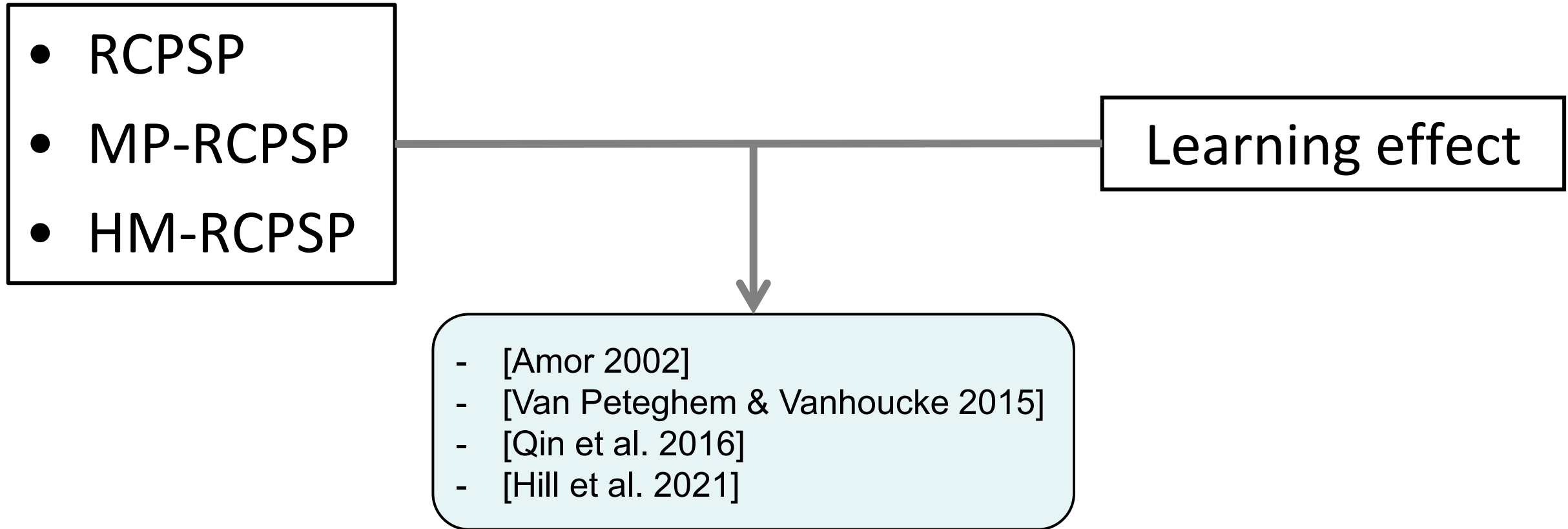
# State of the Art

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- RCPSP
- MP-RCPSP
- HM-RCPSP

Learning effect

# State of the Art



# Objectives

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- Bi-objective lexicographic scheduling problem:
  - 1<sup>st</sup> objective: minimize the sum of projects tardiness
  - 2<sup>nd</sup> objective: minimize the makespan of the entire schedule

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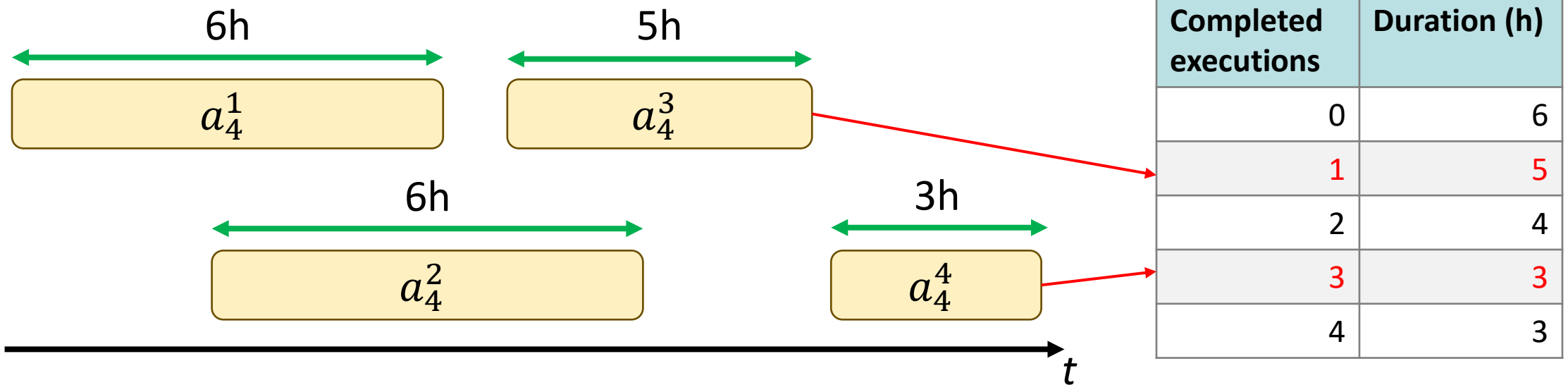
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# Learning Effect Modeling

- Monotonically decreasing function  $dur : \mathbb{N} \rightarrow \mathbb{N}$



## ❖ Problem input:

- Classes, projects and activities
- Due dates
- Learning effect
- Resources

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❖ Solution: a schedule / the start time of each instance of each activity, which satisfies

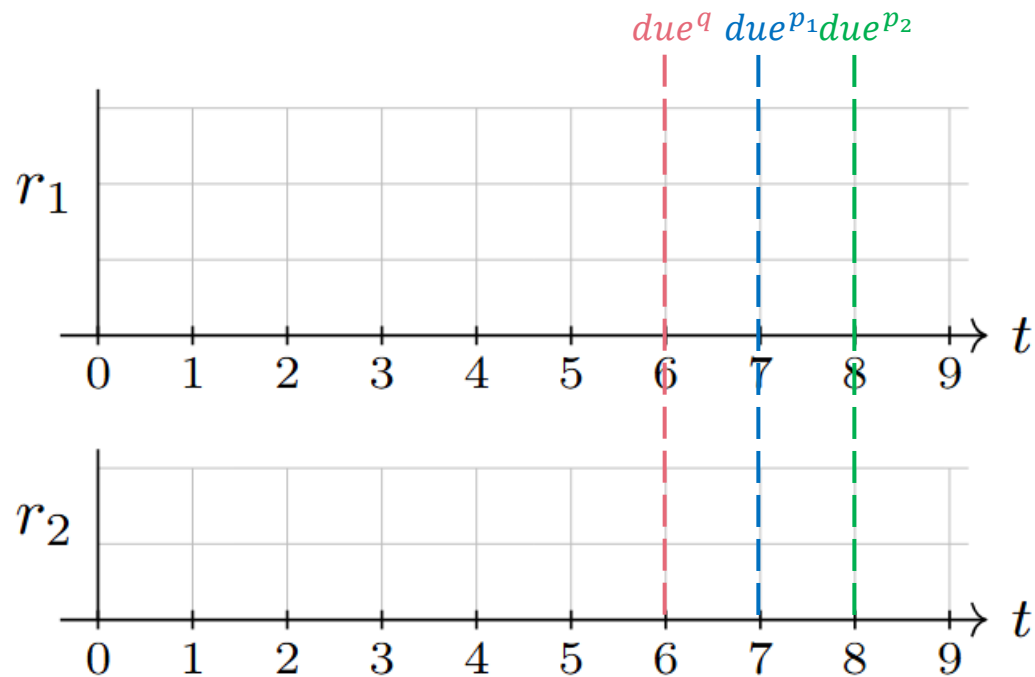
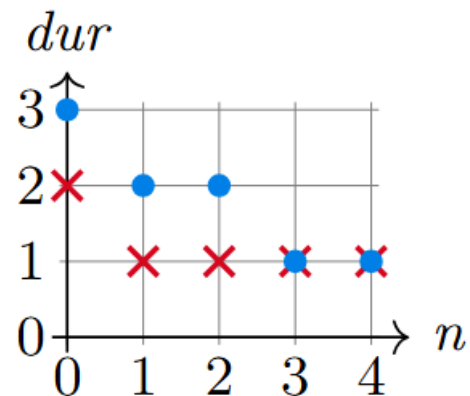
- Precedence relation
- Composition relation
- Resource capacities
- Duration function

- ❖ Problem input:
  - Classes, projects and activities
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- ❖ Solution: a schedule / the start time of each instance of each activity, which satisfies
  - Precedence relation
  - Composition relation
  - Resource capacities
  - Duration function
- ❖ Optimization criteria: sum of tardiness & makespan

# Example

$\mathcal{C}$	$\mathcal{A}_c$	$dur_a$	$r_1$	$r_2$
$c_1$	$a$	$\delta_1$	1	0
	$b$	$\delta_2$	0	1
	$c$	$\delta_1$	0	1
	$d$	-	1	0
$c_2$	$e$	$\delta_2$	0	1
	$f$	$\delta_1$	1	0
	$g$	-	1	0

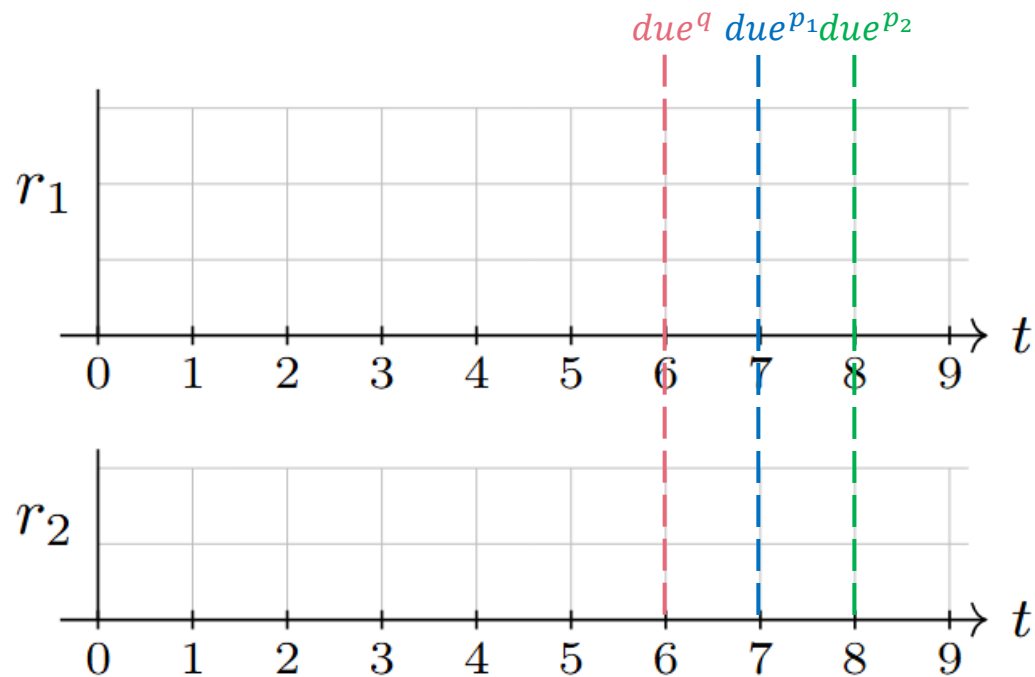
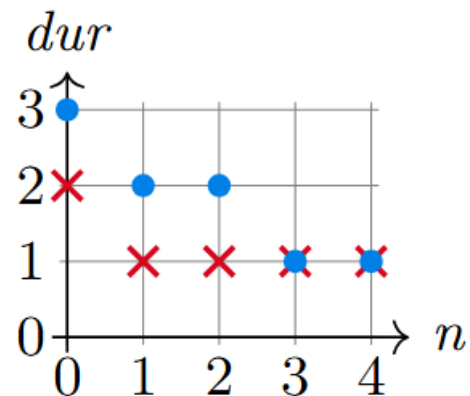
$p_1$   $p_2$



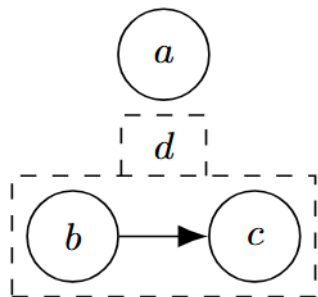
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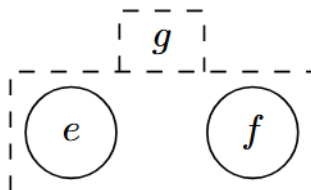
$p_1$   $p_2$



$c_1$  :



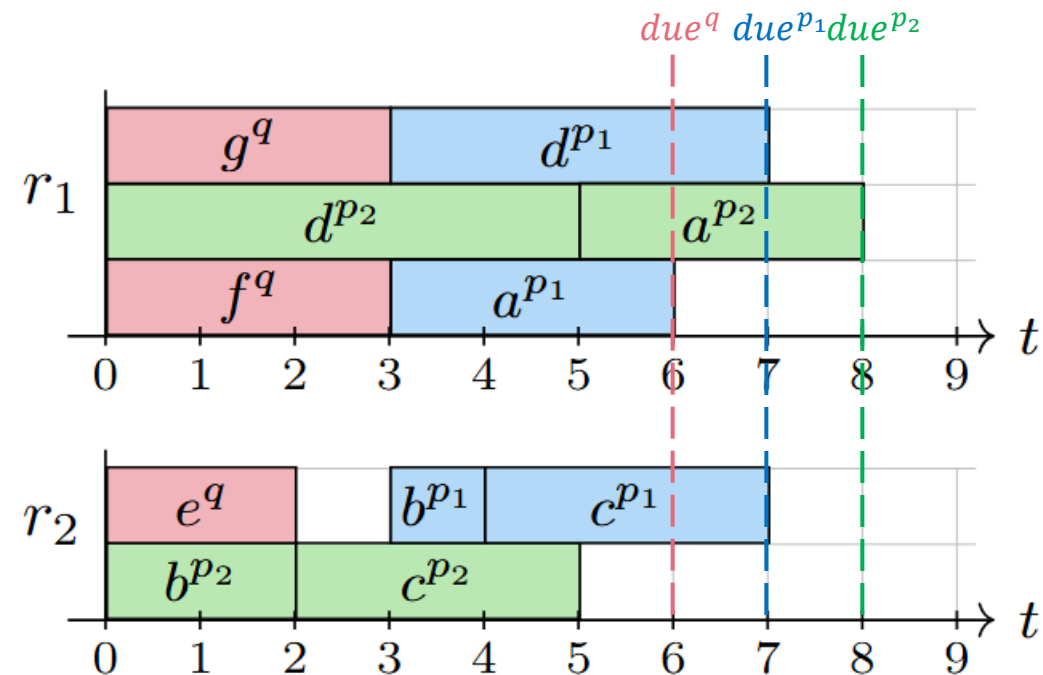
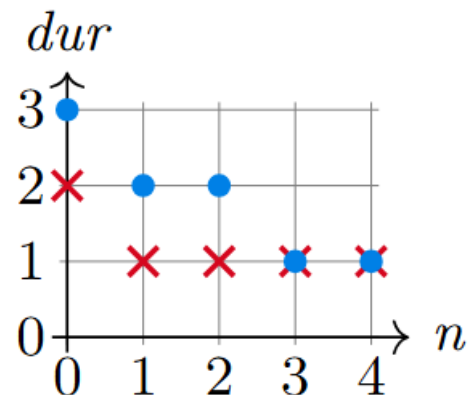
$c_2$  :



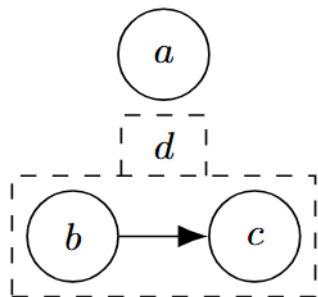
# Example

$\mathcal{C}$	$\mathcal{A}_c$	$dur_a$	$r_1$	$r_2$
$c_1$	$a$	$\delta_1$	1	0
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	$f$	$\delta_1$	1	0
	$g$	-	1	0

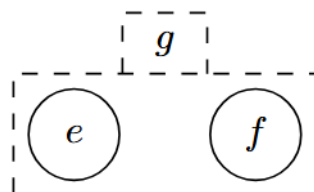
$p_1$   $p_2$



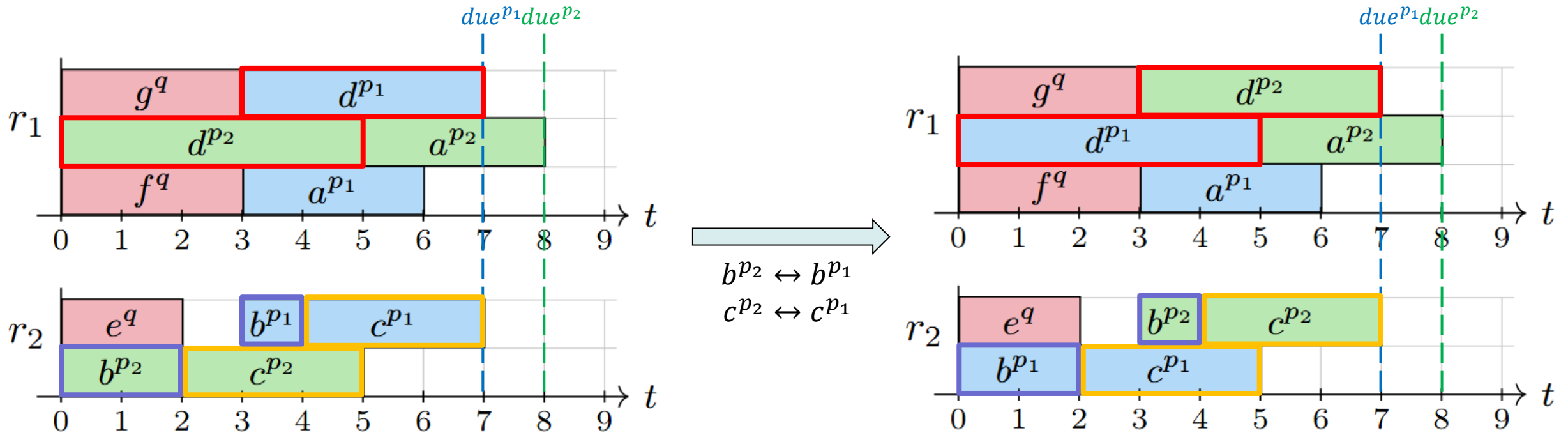
$c_1$  :



$c_2$  :

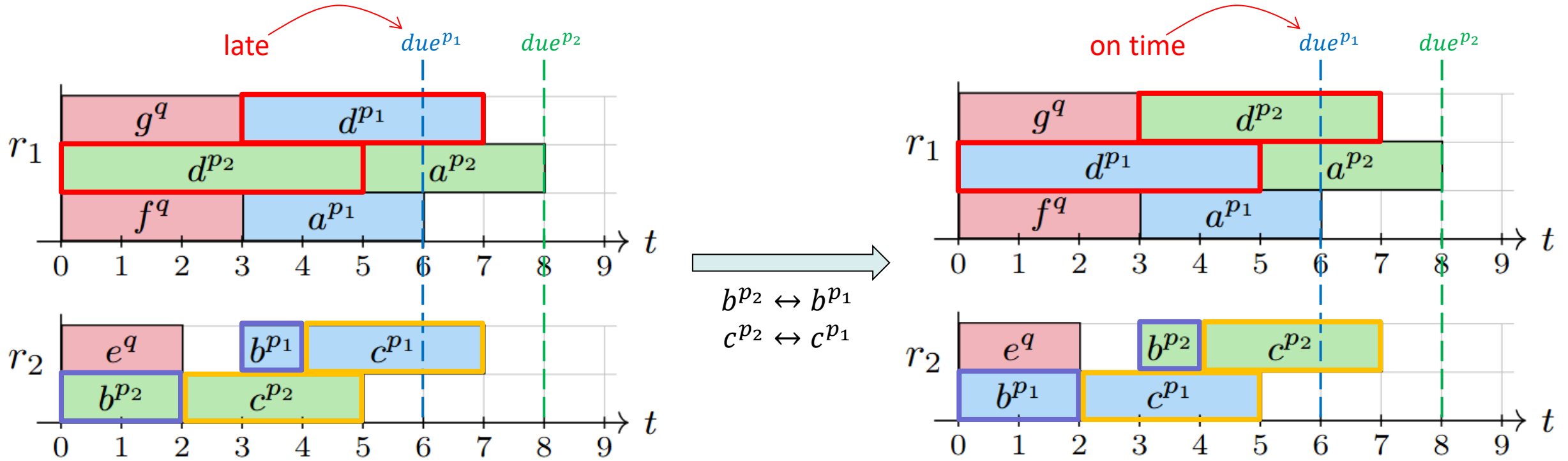


# Symmetry Breaking

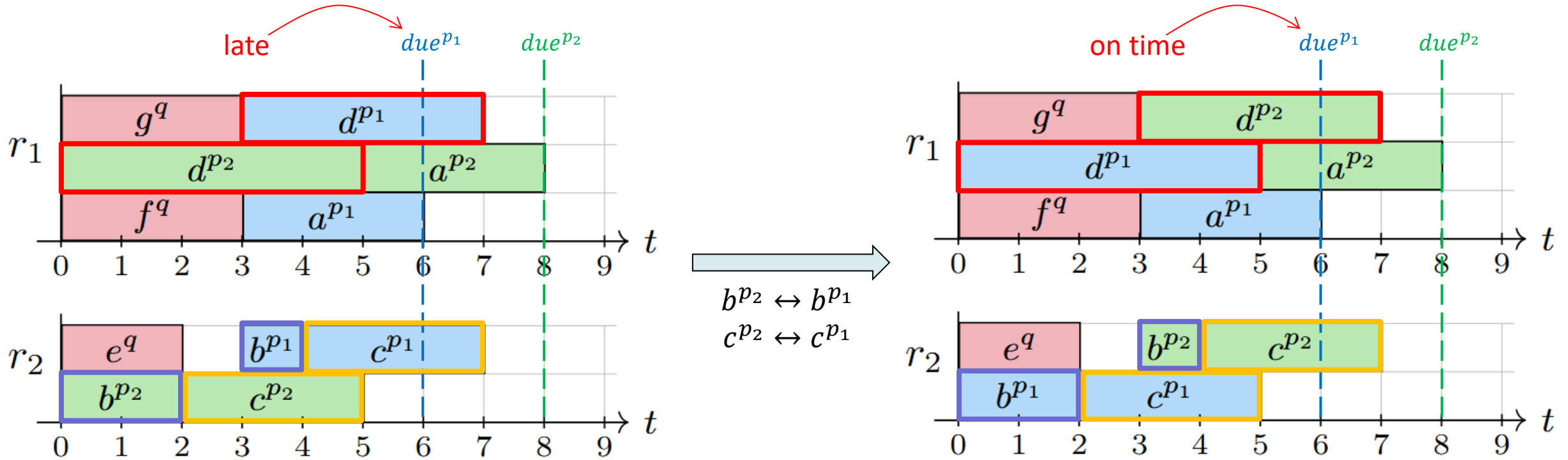




# Symmetry Breaking



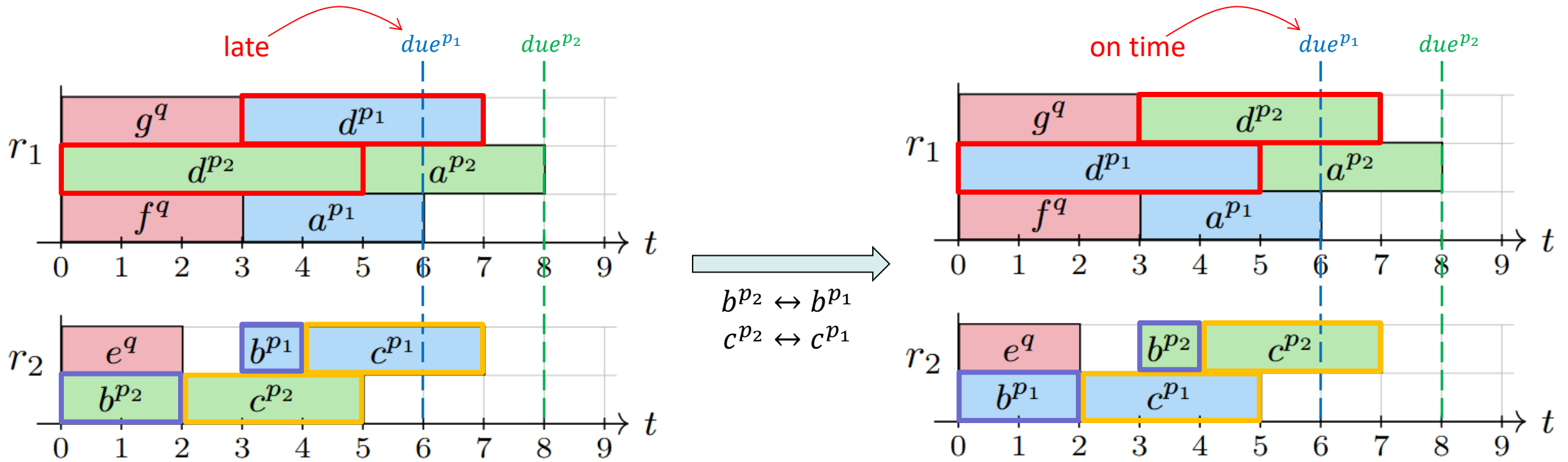
# Symmetry Breaking



## Theorem:

**IF**  $due^{p_1} \leq due^{p_2}$  **THEN**  $criteria(b^{p_1} \text{ before } b^{p_2}) \leq criteria(b^{p_2} \text{ before } b^{p_1})$

# Symmetry Breaking



## Theorem:

**IF**  $due^{p_1} \leq due^{p_2}$  **THEN**  $criteria(b^{p_1} \text{ before } b^{p_2}) \leq criteria(b^{p_2} \text{ before } b^{p_1})$



Activities are performed in order of due date for same-class projects

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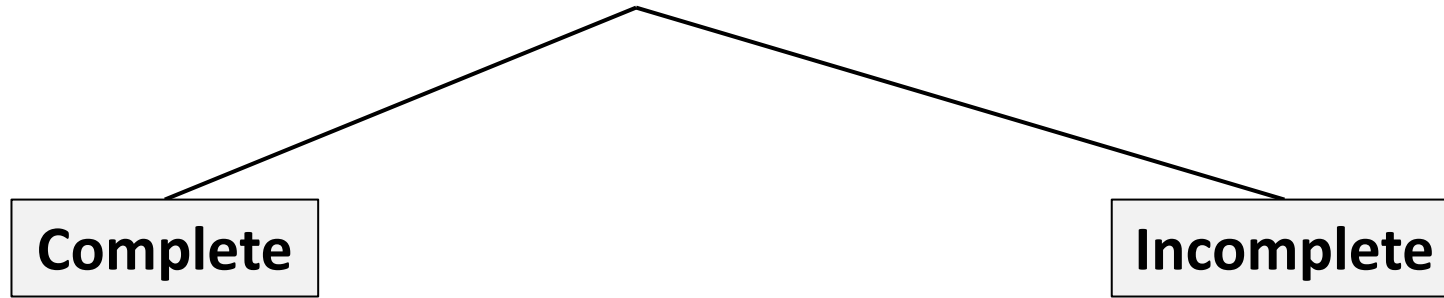
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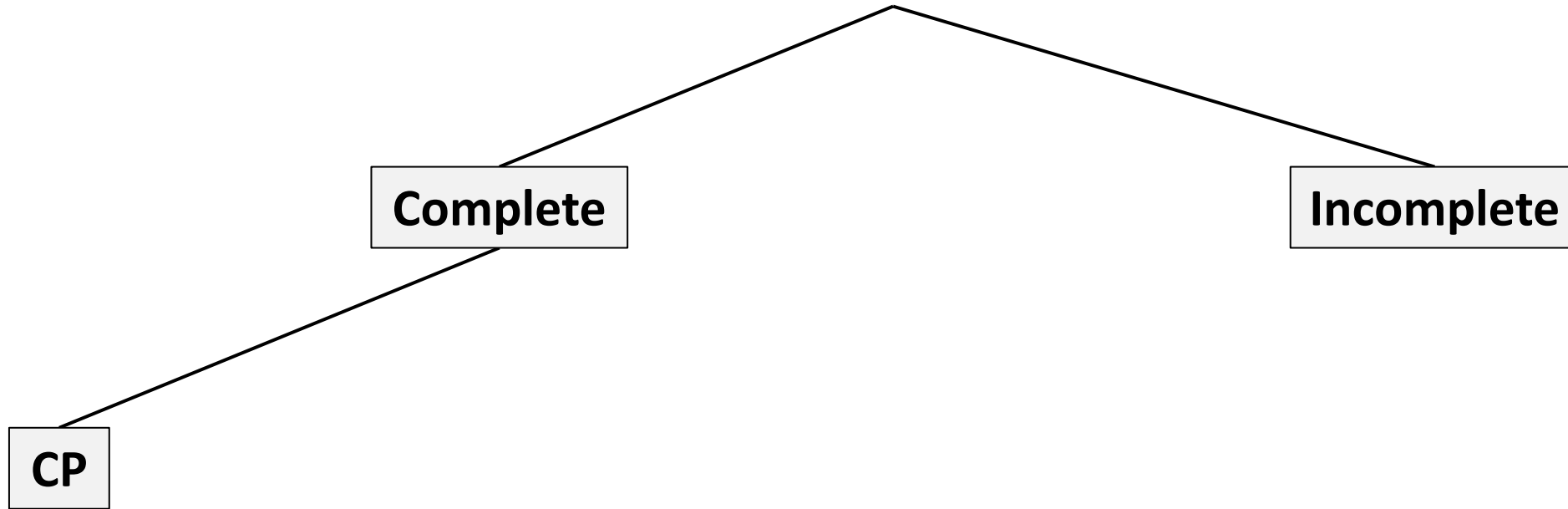
# Solving Approaches

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# Solving Approaches

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# Constraint Programming

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For each activity  $a$  of each project  $p$ :

- Interval variable  $\mathbf{itv}_{a,p}$  contains properties of this activity instance (starting time, duration)
- Integer variable  $\mathbf{n}_{a,p}$  indicates the number of executions that have already been completed of this activity instance

# Constraint Programming

$$\text{minimize} \left( \underbrace{\sum_{c \in C, p \in I_c, a \in A_c} \max(0, \text{endOf}(\text{itv}_{a,p}) - \text{due}_p)}_{\text{Tardiness}}, \underbrace{\max_{c \in C, p \in I_c, a \in A_c} \text{endOf}(\text{itv}_{a,p})}_{\text{Makespan}} \right)$$



# Constraint Programming

$$\text{minimize} \left( \underbrace{\sum_{c \in C, p \in I_c, a \in A_c} \max(0, \text{endOf}(\text{itv}_{a,p}) - \text{due}_p)}_{\text{Tardiness}}, \underbrace{\max_{c \in C, p \in I_c, a \in A_c} \text{endOf}(\text{itv}_{a,p})}_{\text{Makespan}} \right)$$

Classical  
RCPSP

$$\left\{ \begin{array}{l} \forall r \in R \quad \sum_{c \in C, p \in I_c, a \in A_c} \text{pulse}(\text{itv}_{a,p}, \text{cons}_{r,a}) \leq \text{capa}_r \quad (1) \\ \forall c \in C, \forall p \in I_c, \forall (a, b) \in \text{Prec}_c \quad \text{endBeforeStart}(\text{itv}_{a,p}, \text{itv}_{b,p}) \quad (2) \end{array} \right.$$

# Constraint Programming

$$\text{minimize} \left( \underbrace{\sum_{c \in C, p \in I_c, a \in A_c} \max(0, \text{endOf}(\text{itv}_{a,p}) - \text{due}_p)}_{\text{Tardiness}}, \underbrace{\max_{c \in C, p \in I_c, a \in A_c} \text{endOf}(\text{itv}_{a,p})}_{\text{Makespan}} \right)$$

Classical  
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$$\forall r \in R$$

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$$\forall c \in C, \forall p \in I_c, \forall (a, b) \in \text{Prec}_c$$

$$\text{endBeforeStart}(\text{itv}_{a,p}, \text{itv}_{b,p}) \quad (2)$$

Compound  
Activities

$$\forall c \in C, \forall p \in I_c, \forall a \in A_c^{\text{Comp}}$$

$$\text{span}(\text{itv}_{a,p}, \{\text{itv}_{b,p} \mid (a, b) \in \text{Comp}_c\}) \quad (3)$$

# Constraint Programming

$$\text{minimize} \left( \underbrace{\sum_{c \in C, p \in I_c, a \in A_c} \max(0, \text{endOf}(\text{itv}_{a,p}) - \text{due}_p)}_{\text{Tardiness}}, \underbrace{\max_{c \in C, p \in I_c, a \in A_c} \text{endOf}(\text{itv}_{a,p})}_{\text{Makespan}} \right)$$

Classical  
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$$\forall r \in R$$

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$$\text{endBeforeStart}(\text{itv}_{a,p}, \text{itv}_{b,p}) \quad (2)$$

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$$\forall c \in C, \forall p \in I_c, \forall a \in A_c^{\text{Comp}}$$

$$\text{span}(\text{itv}_{a,p}, \{\text{itv}_{b,p} \mid (a, b) \in \text{Comp}_c\}) \quad (3)$$

Learning  
Effect

$$\forall c \in C, \forall p \in I_c, \forall a \in A_c^{\text{Atom}}$$

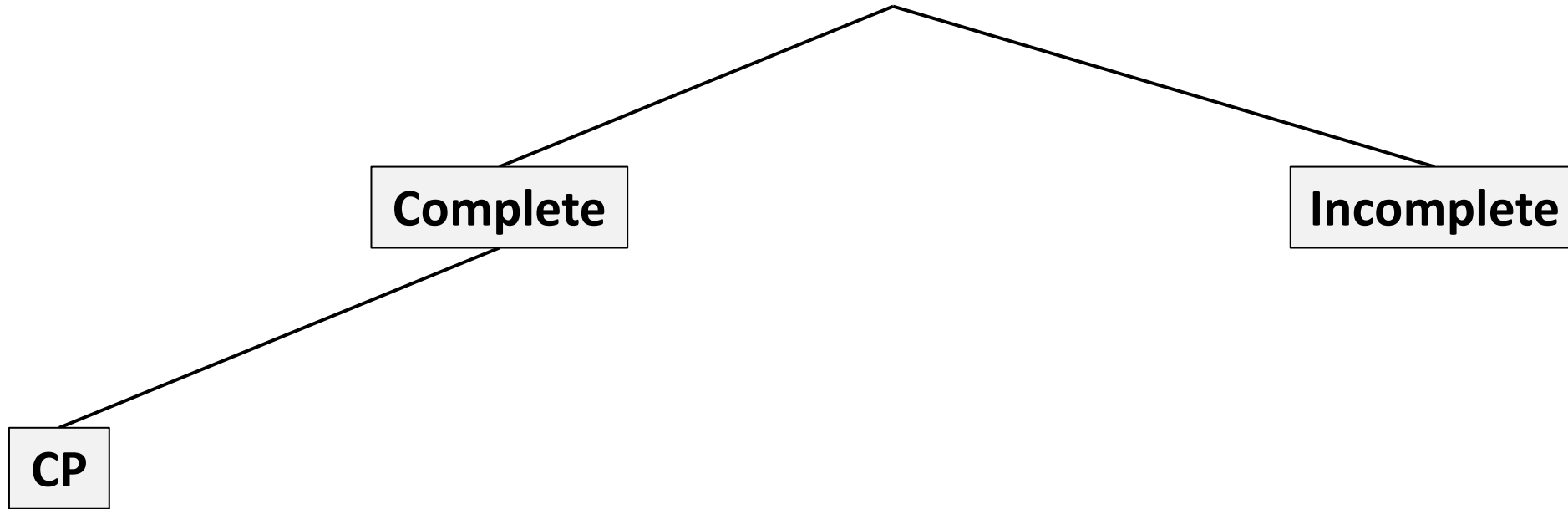
$$\mathbf{n}_{a,p} = \sum_{q \in I_c} (\text{endOf}(\text{itv}_{a,q}) \leq \text{startOf}(\text{itv}_{a,p})) \quad (4)$$

$$\forall c \in C, \forall p \in I_c, \forall a \in A_c^{\text{Atom}}$$

$$\text{lengthOf}(\text{itv}_{a,p}) = \text{dur}_a(\mathbf{n}_{a,p}) \quad (5)$$

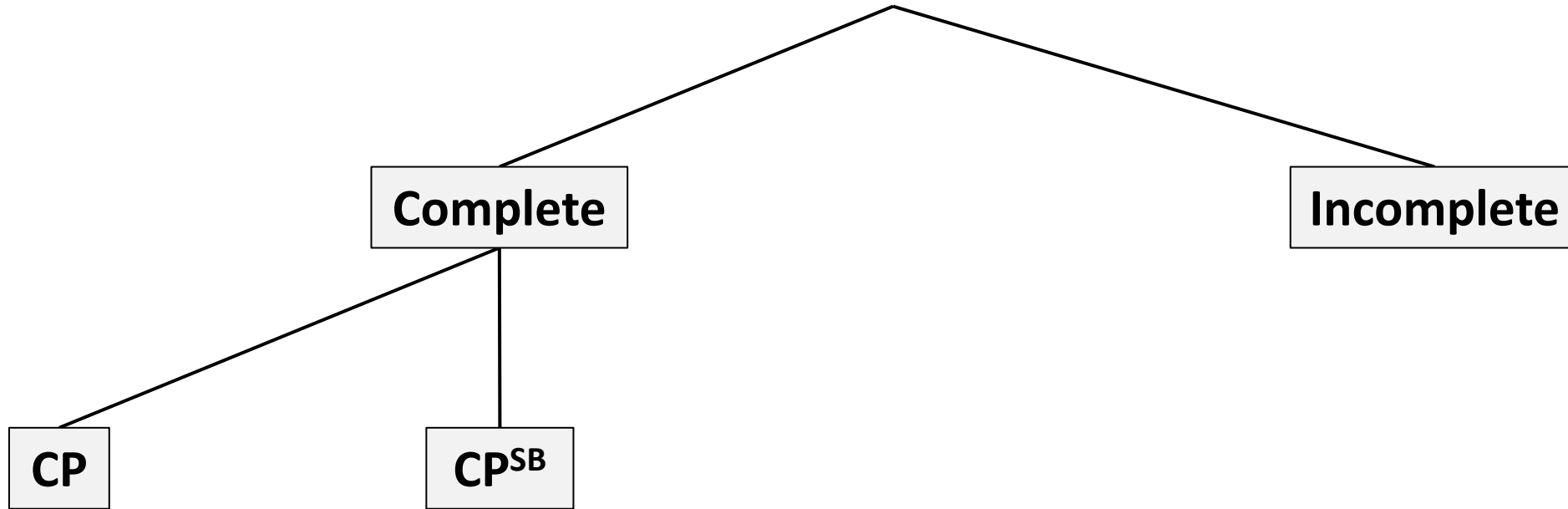
# Solving Approaches

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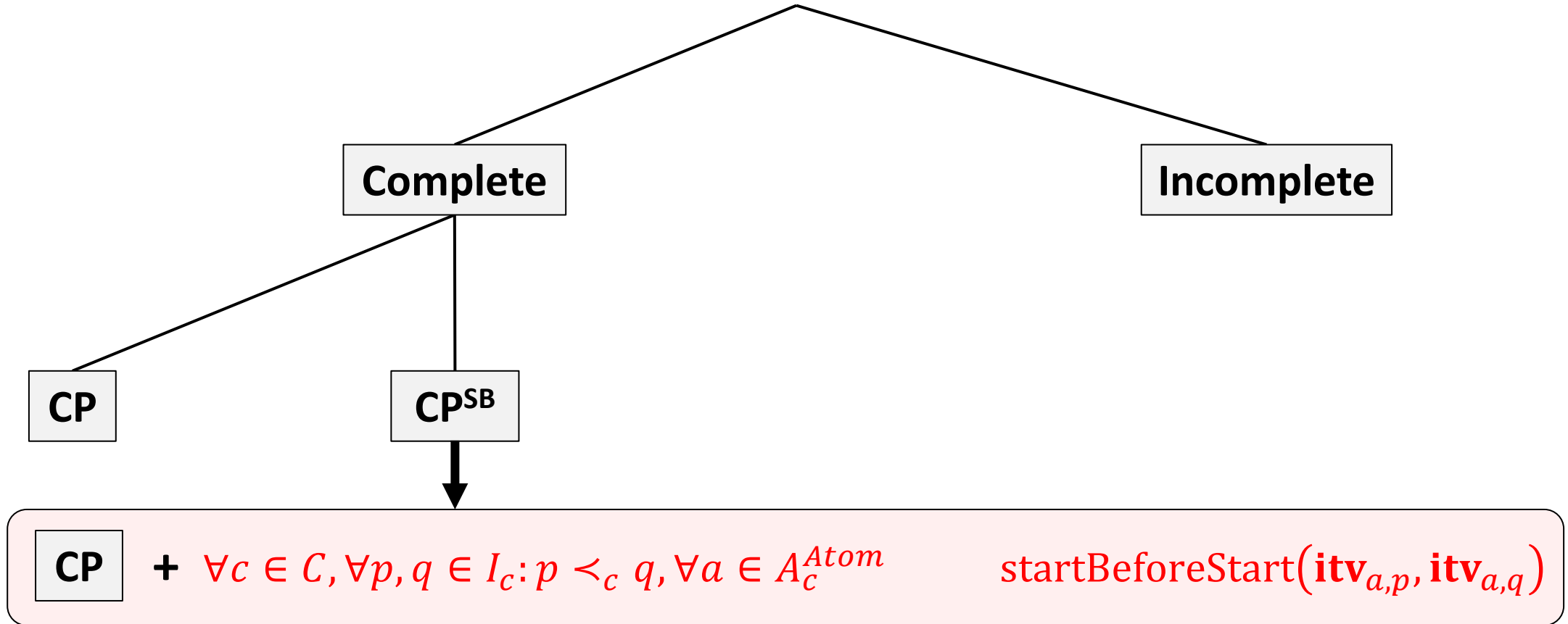


# Solving Approaches

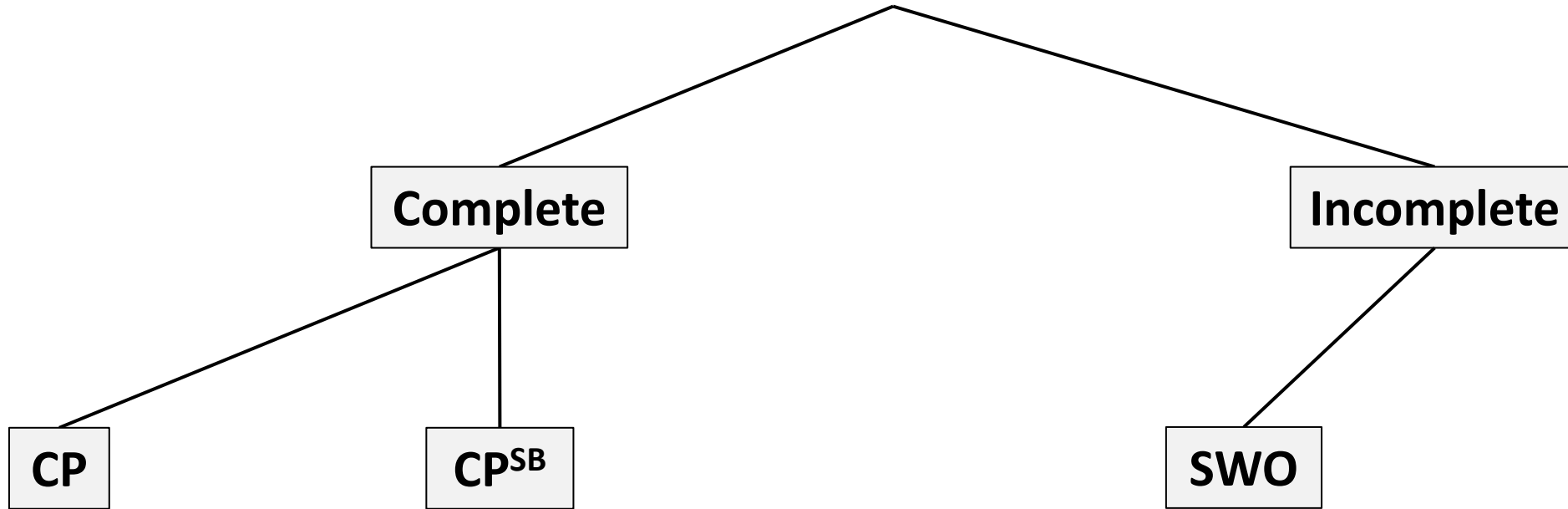
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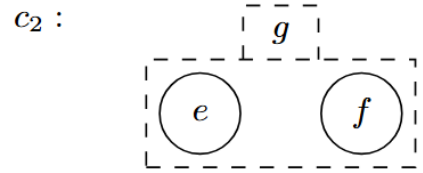
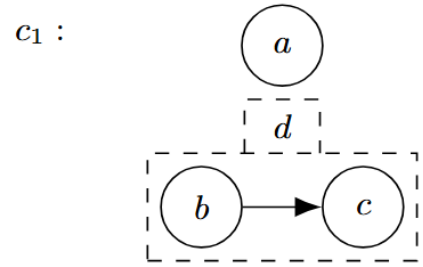
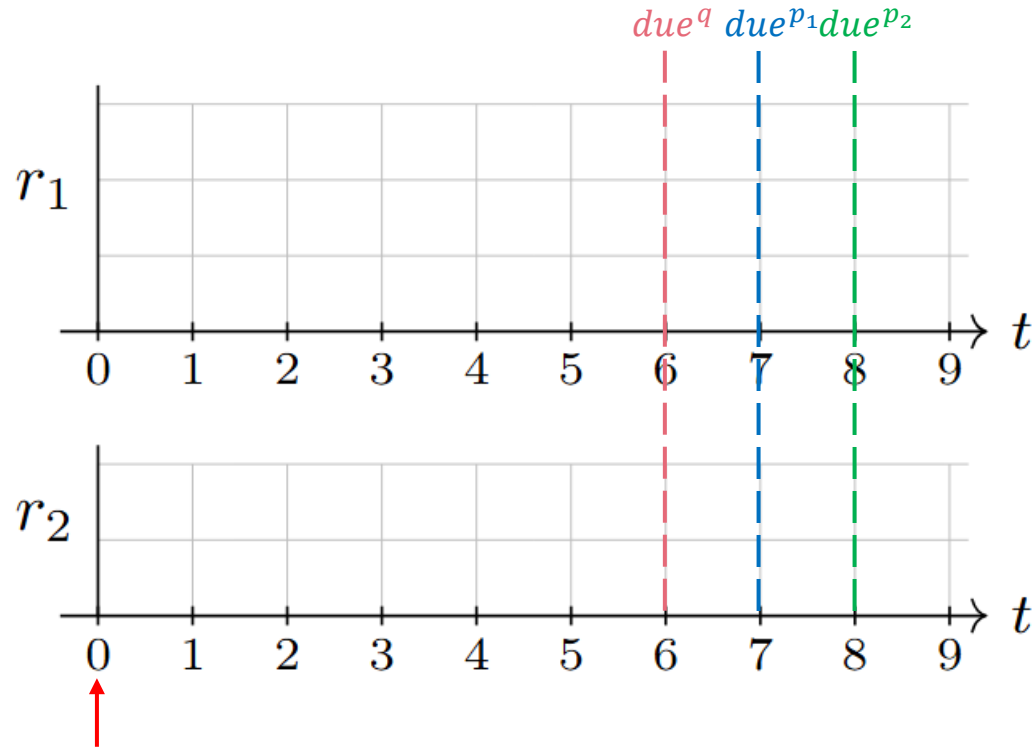
# Solving Approaches



# Solving Approaches

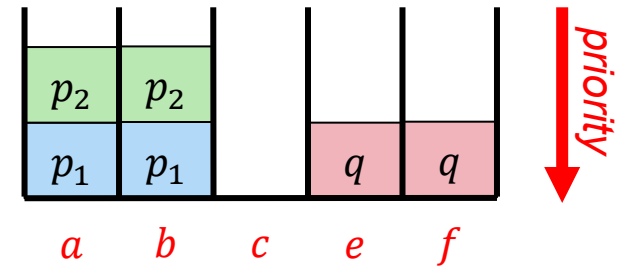
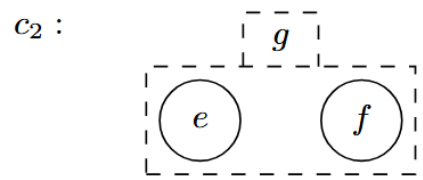
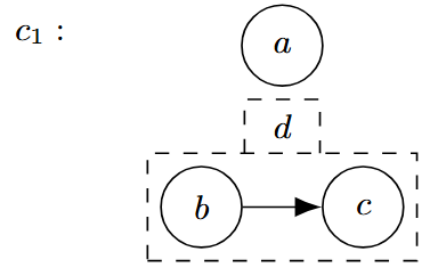
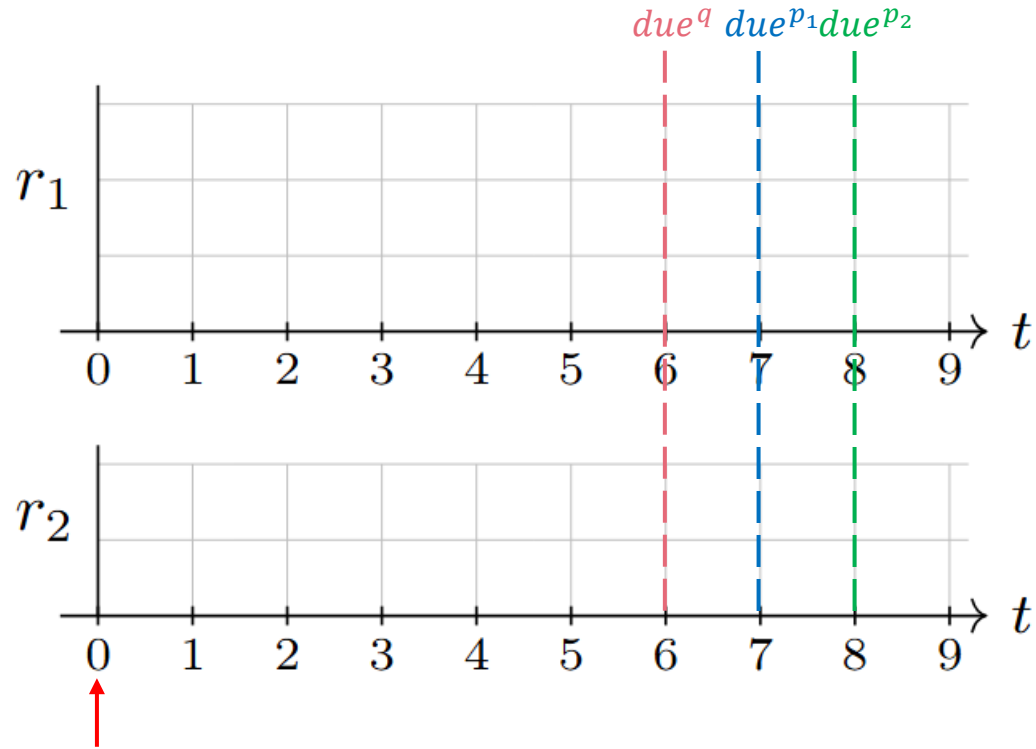


# SWO (Greedy Algorithm)

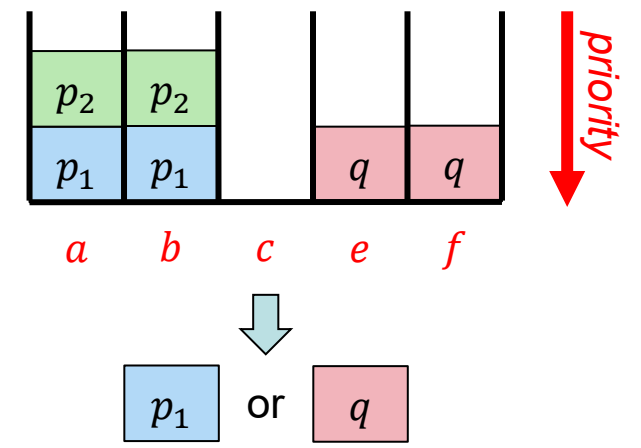
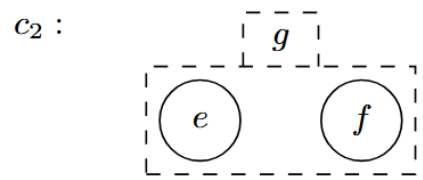
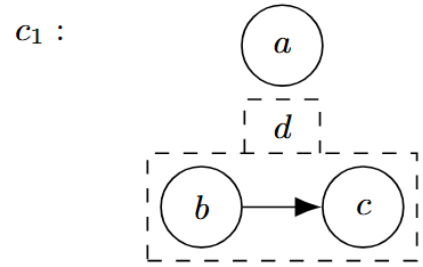
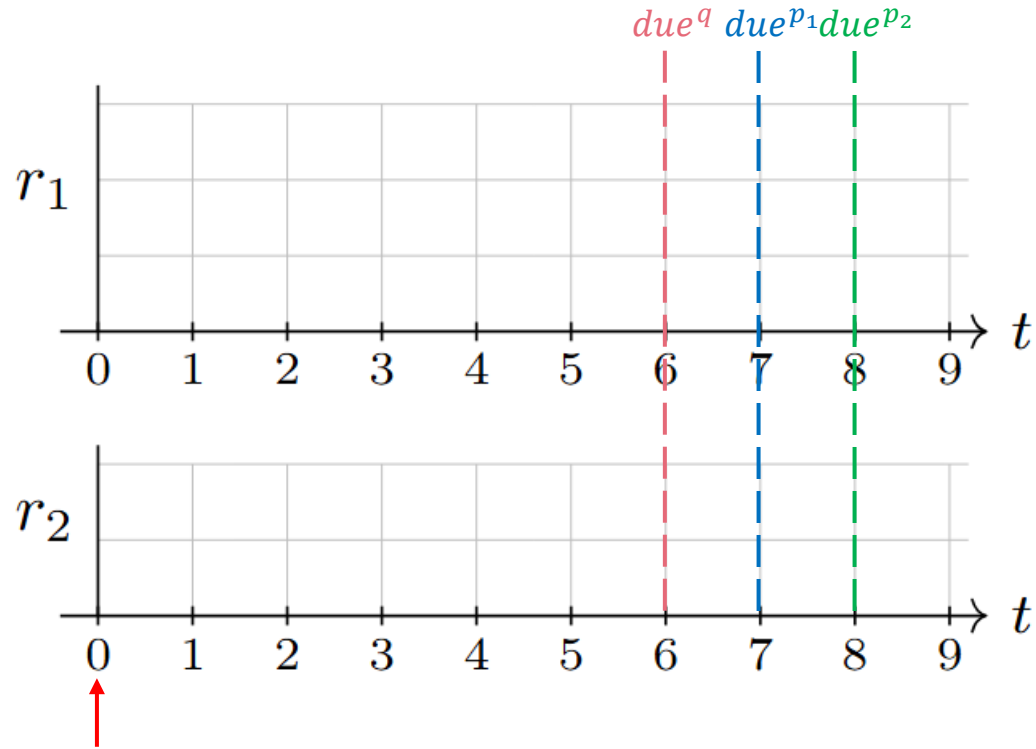




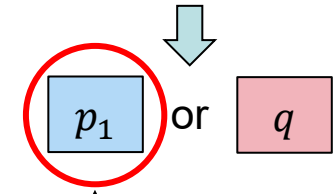
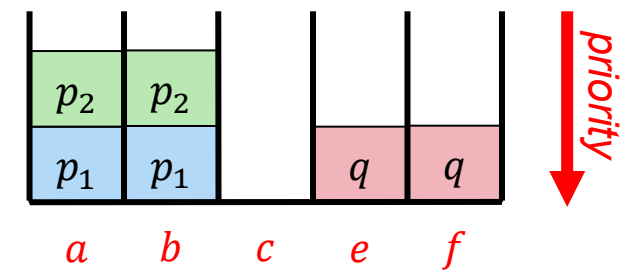
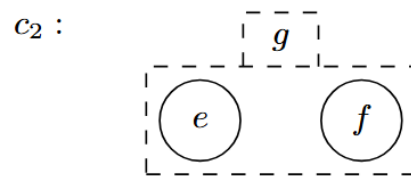
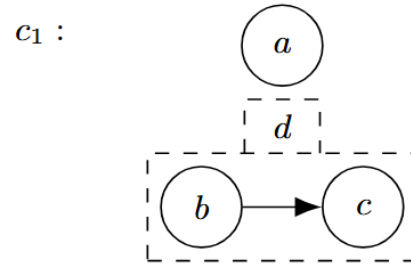
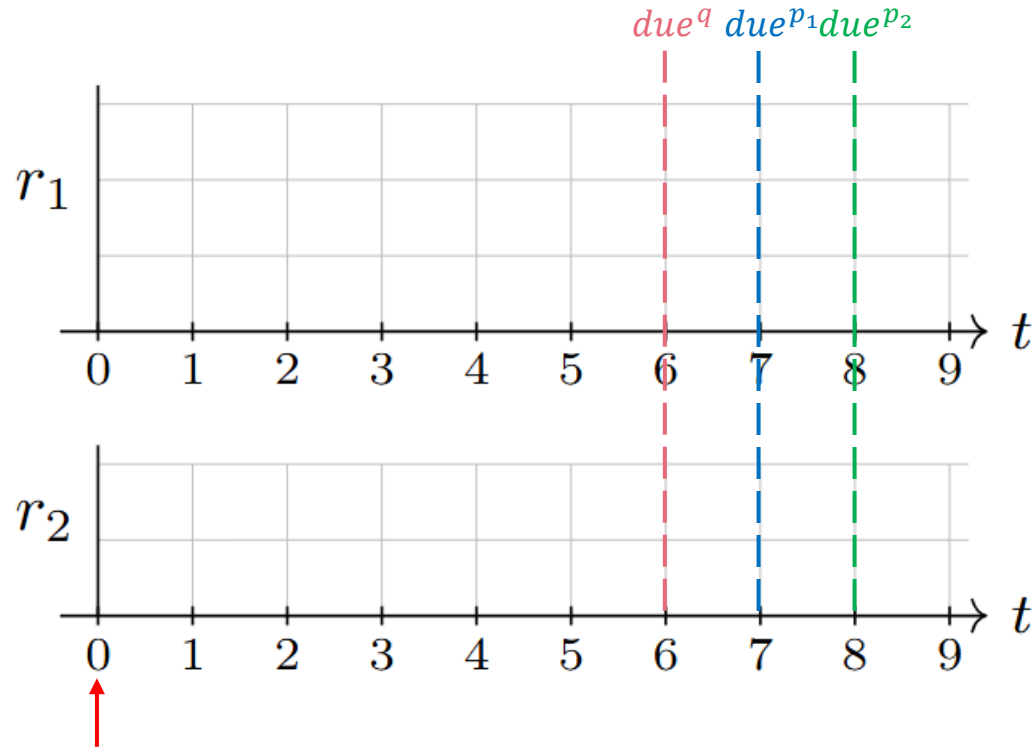
# SWO (Greedy Algorithm)



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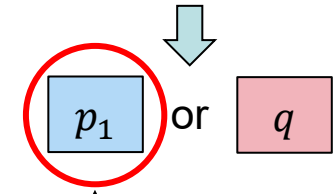
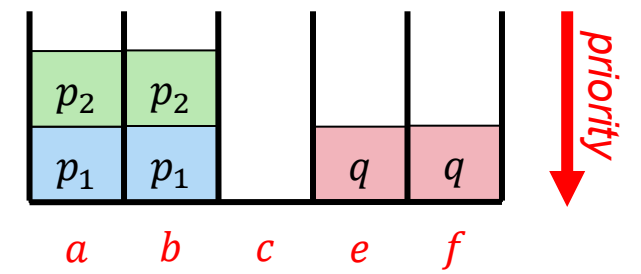
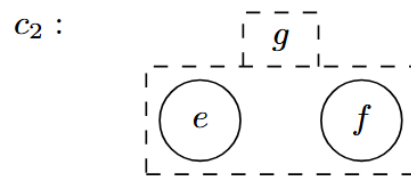
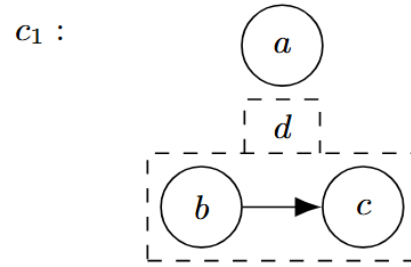
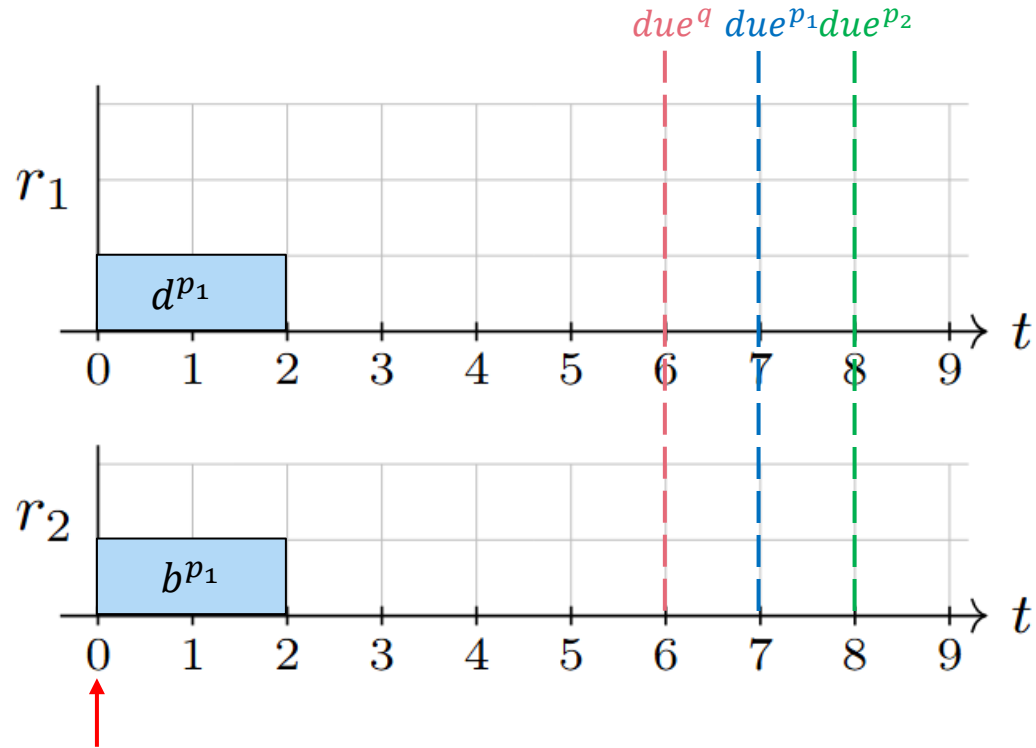


# SWO (Greedy Algorithm)



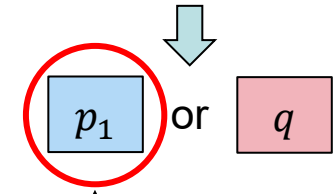
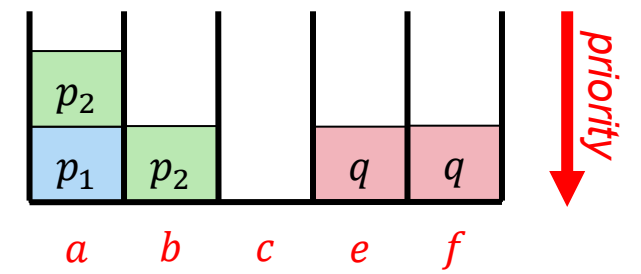
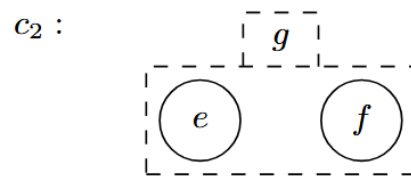
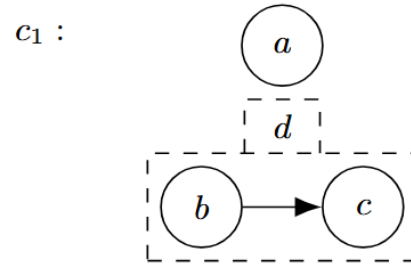
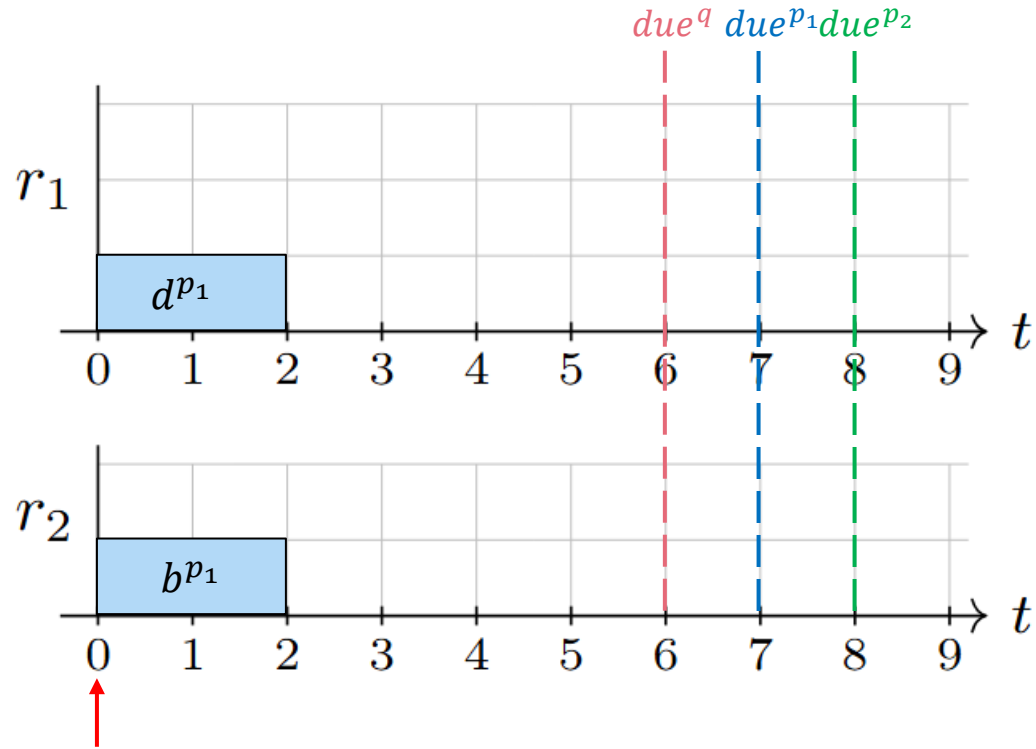
- Due date
- Required time to complete
- Current time moment

# SWO (Greedy Algorithm)



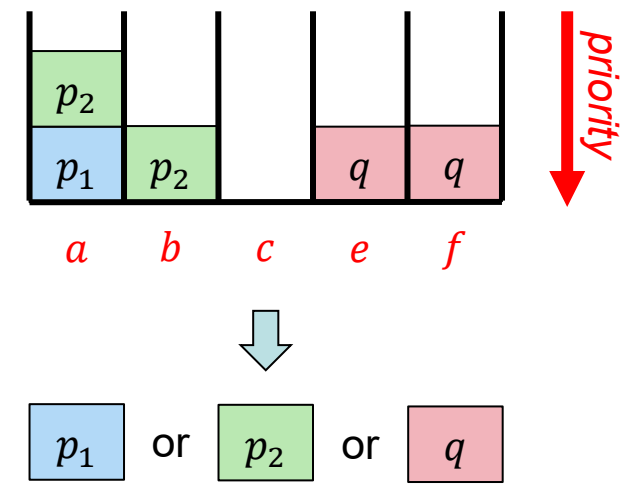
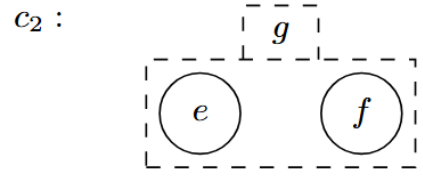
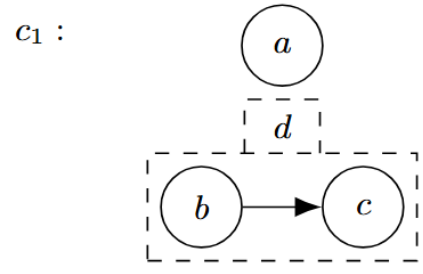
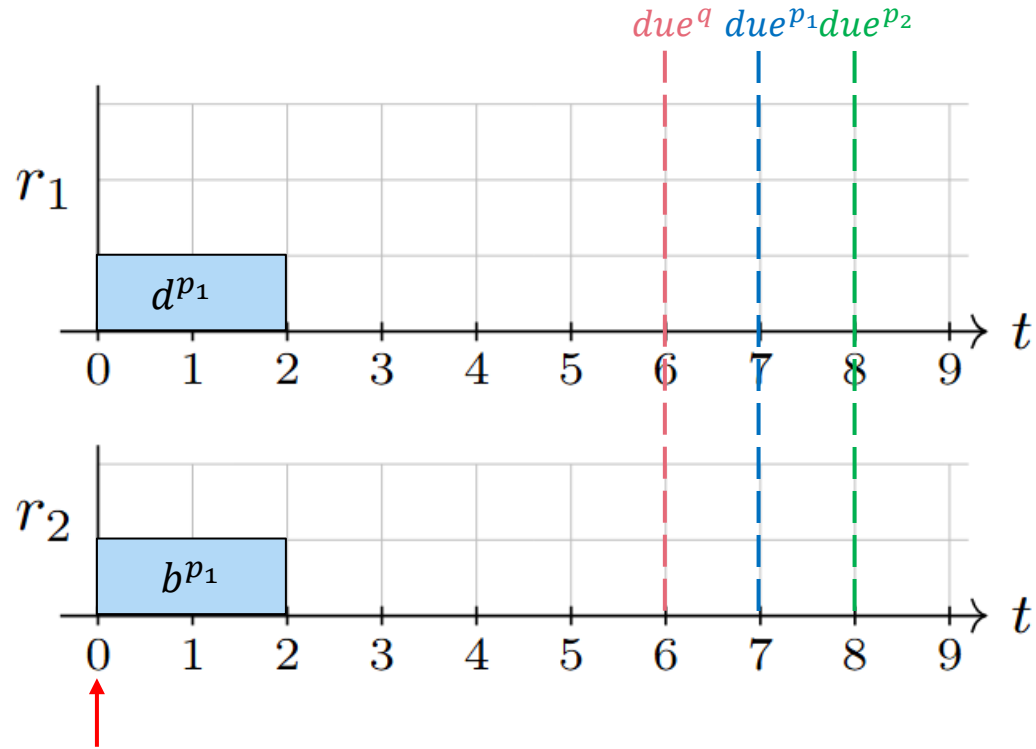
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# SWO (Greedy Algorithm)

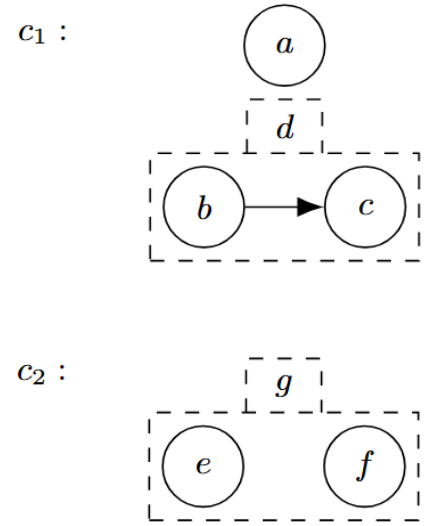
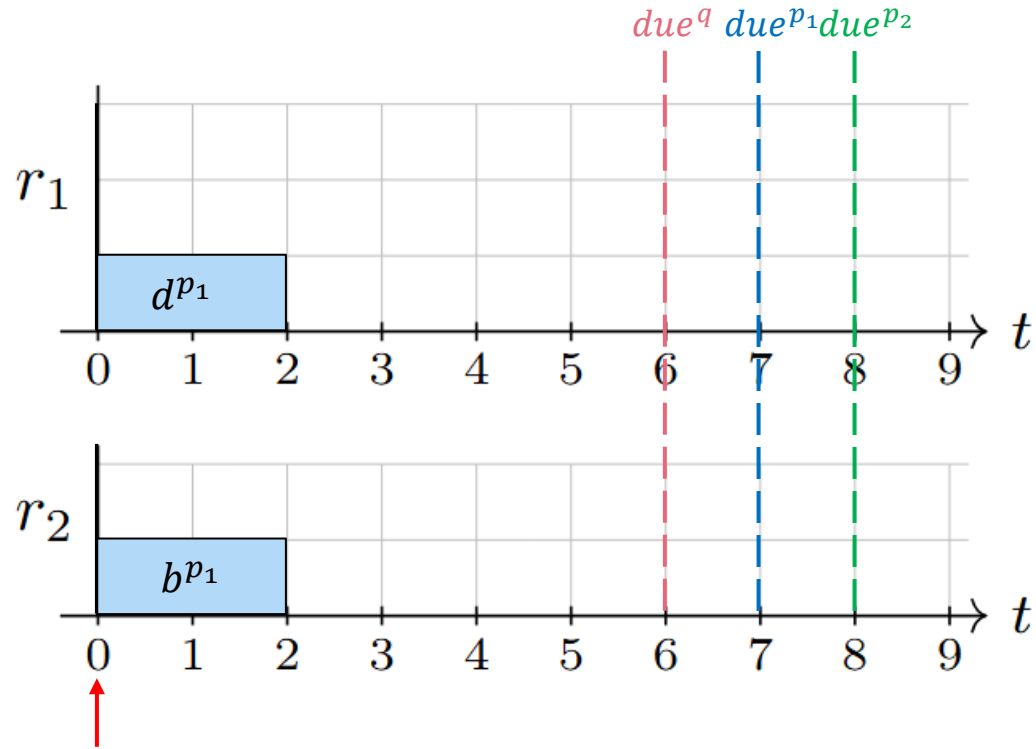


- Due date
- Required time to complete
- Current time moment

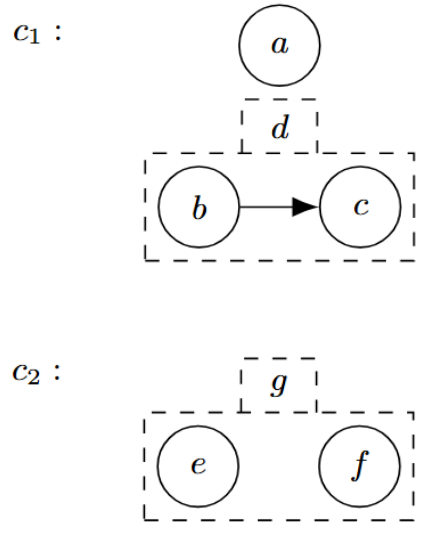
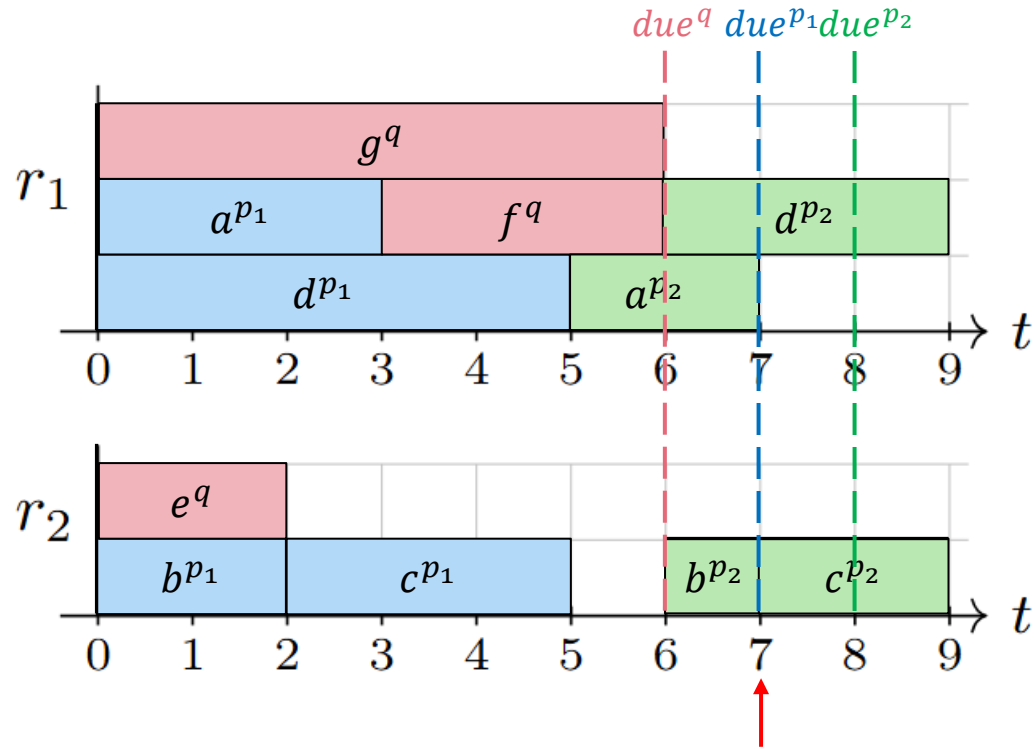
# SWO (Greedy Algorithm)



# SWO (Greedy Algorithm)

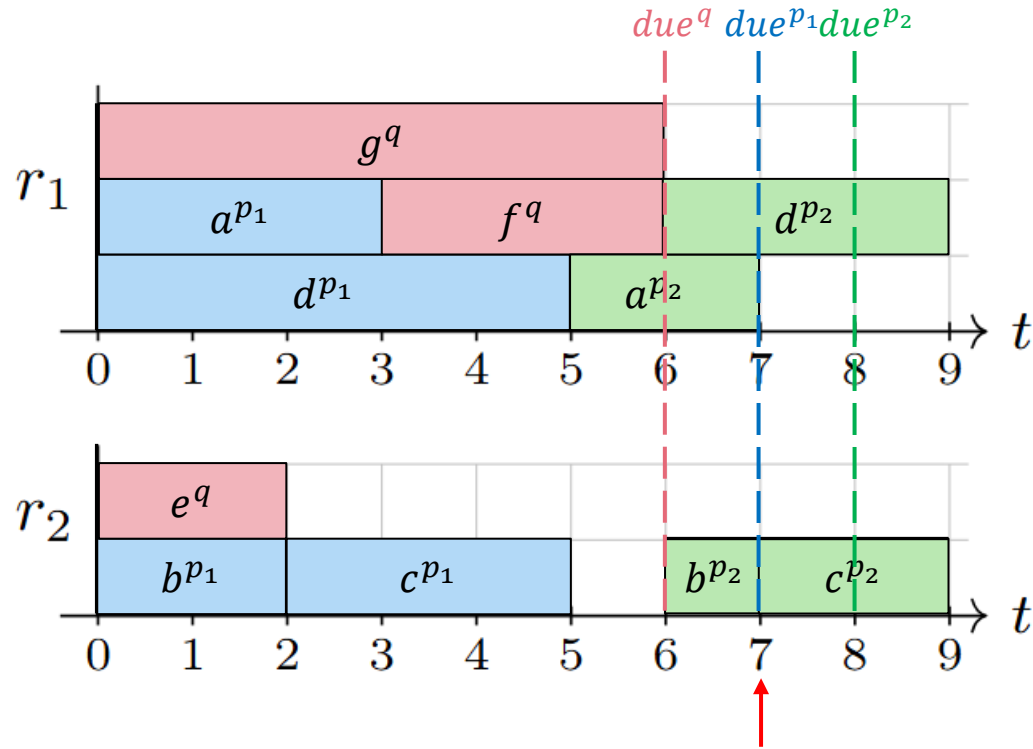


# SWO (Greedy Algorithm)

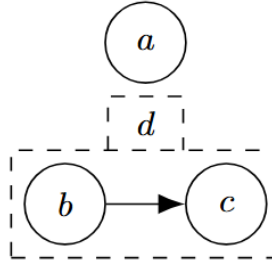




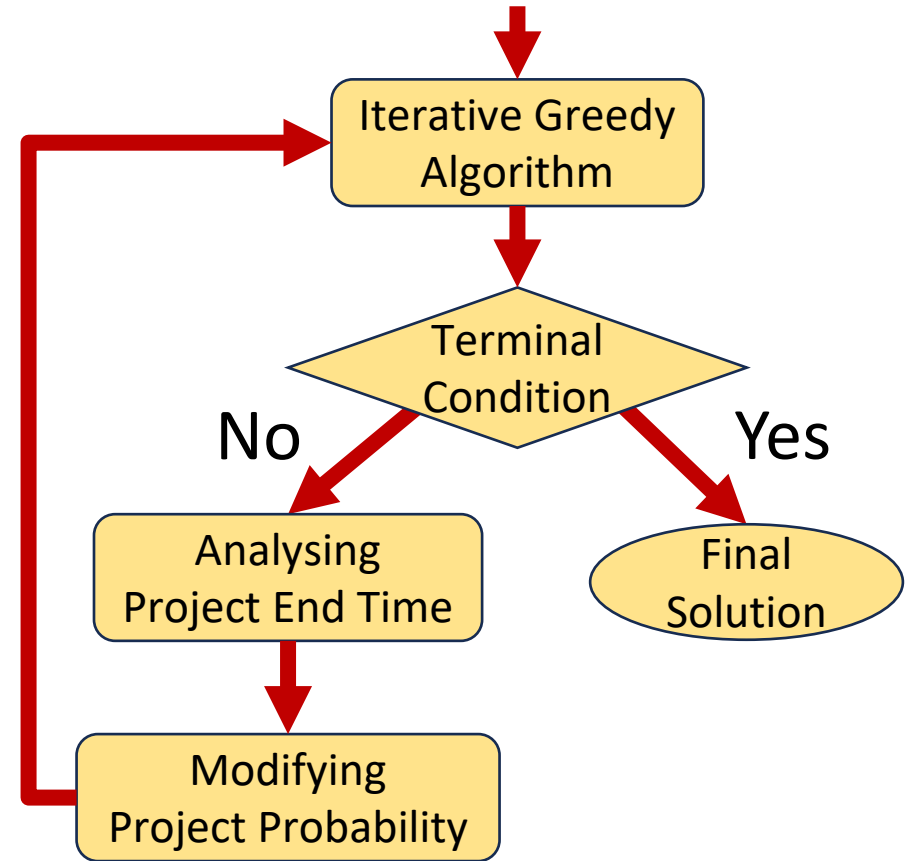
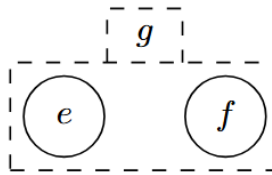
# SWO (Greedy Algorithm)



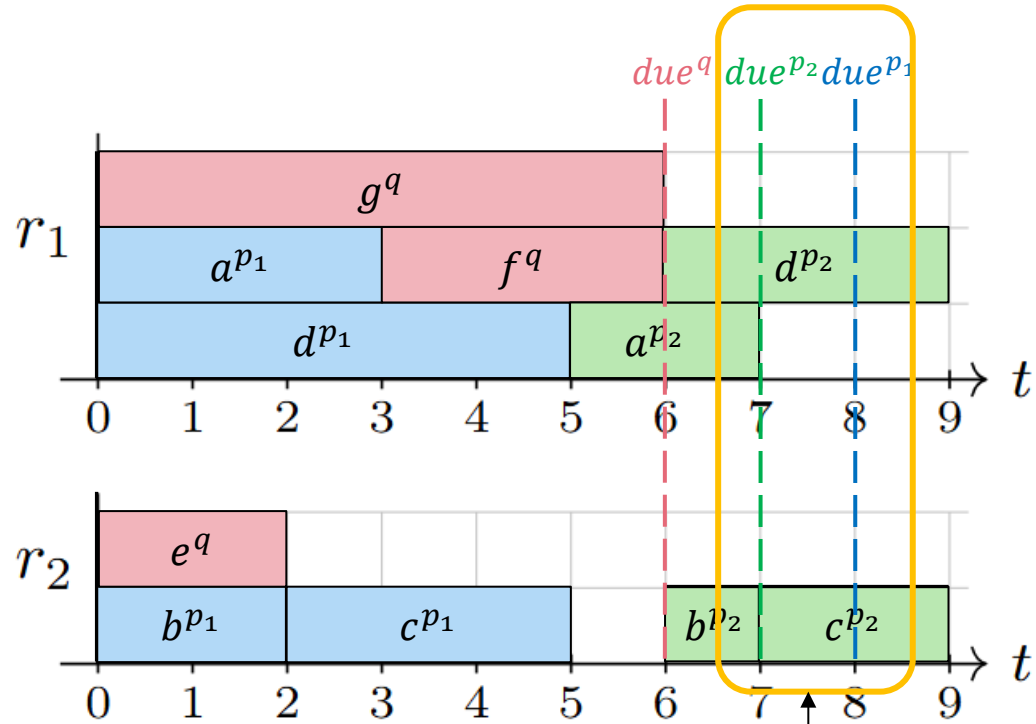
$c_1 :$



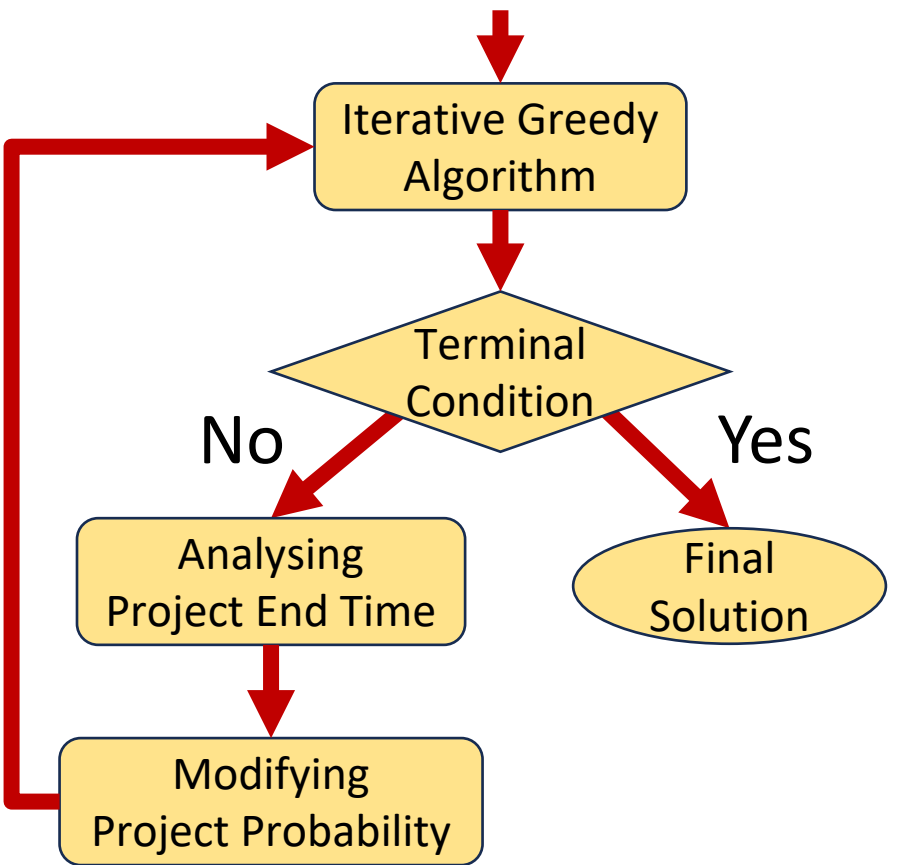
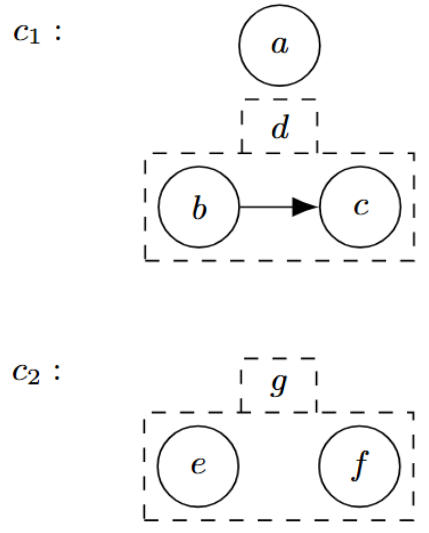
$c_2 :$



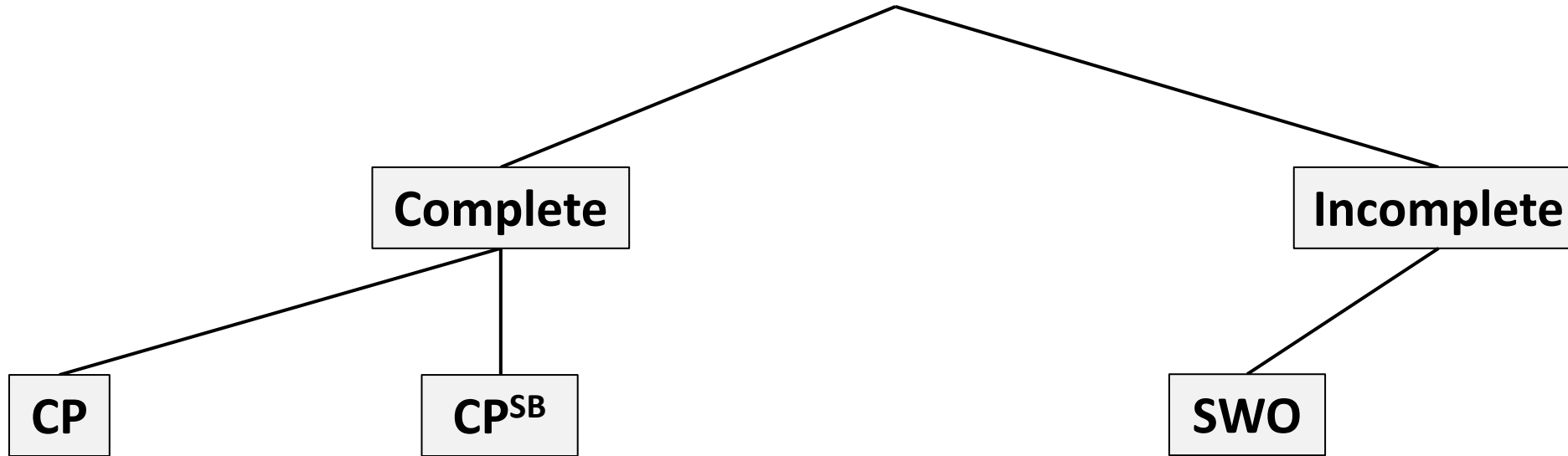
# SWO (Greedy Algorithm)



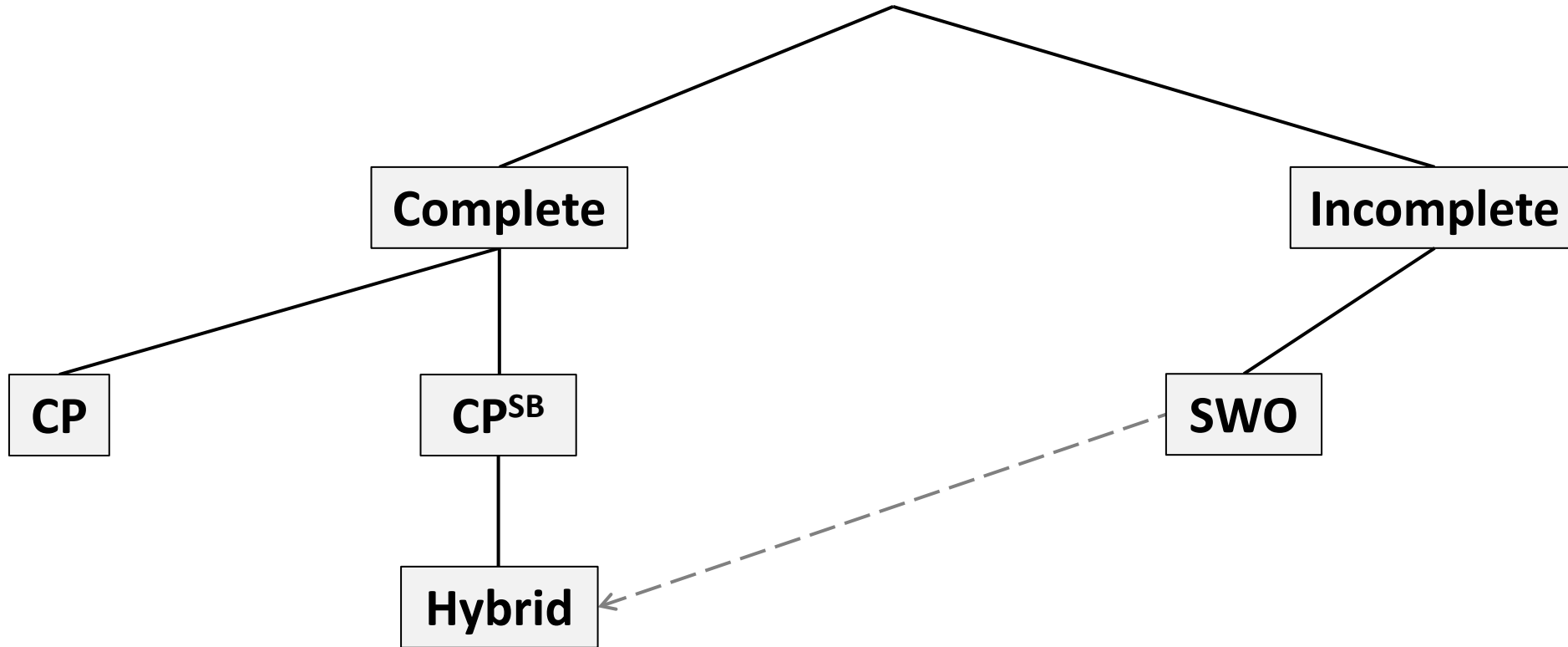
artificial due date - only used to calculate the probabilities



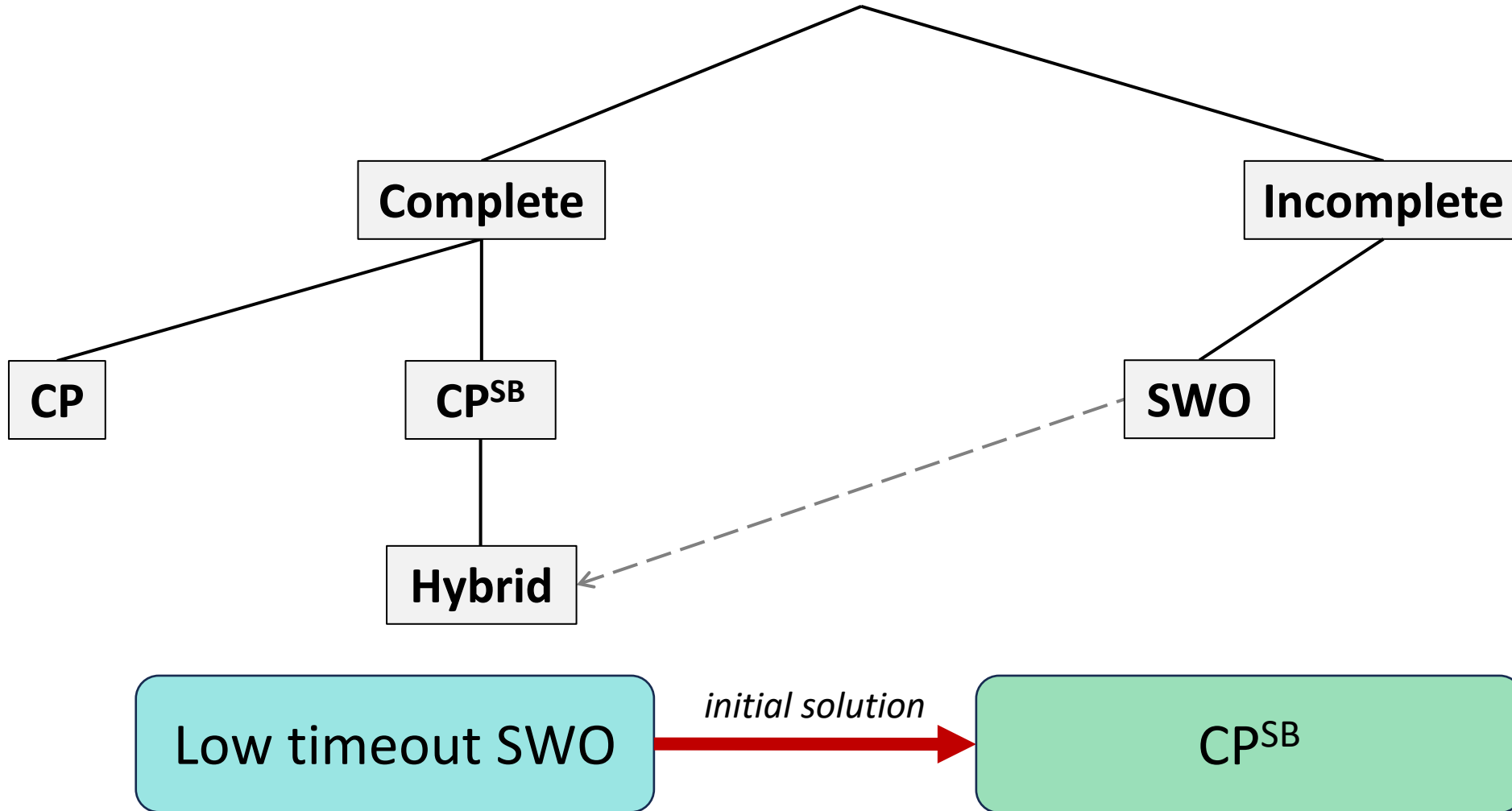
# Solving Approaches



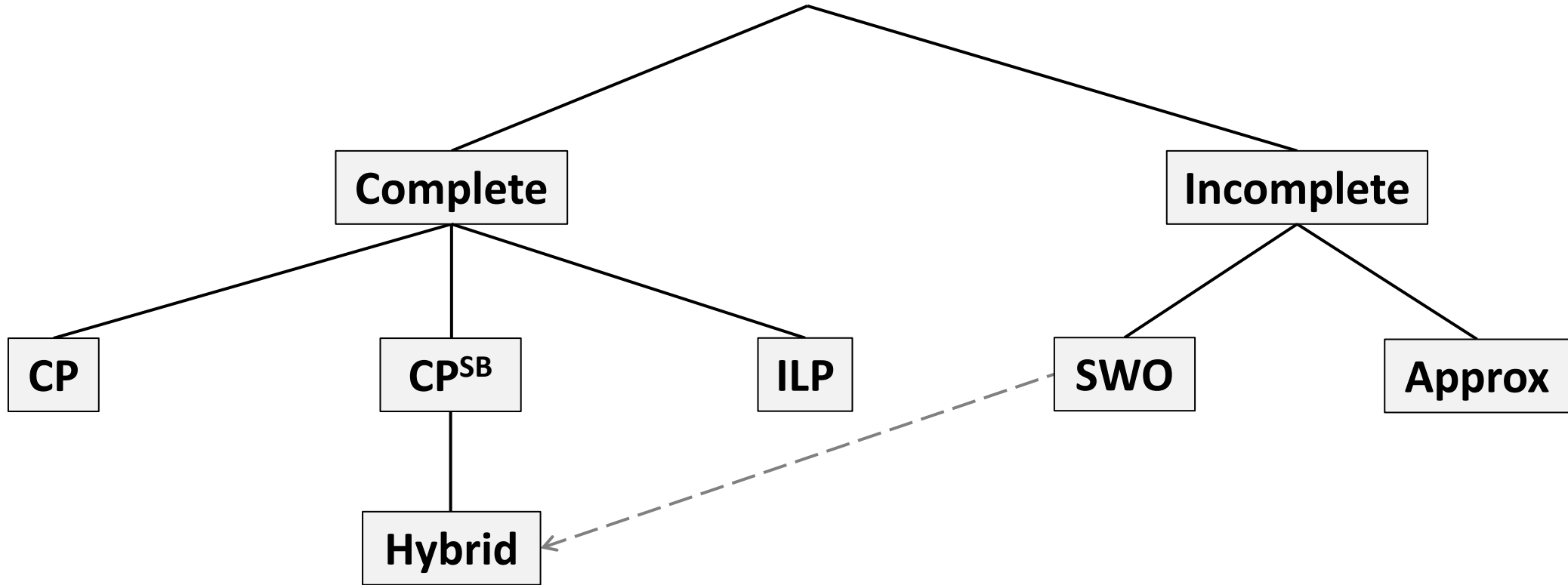
# Solving Approaches



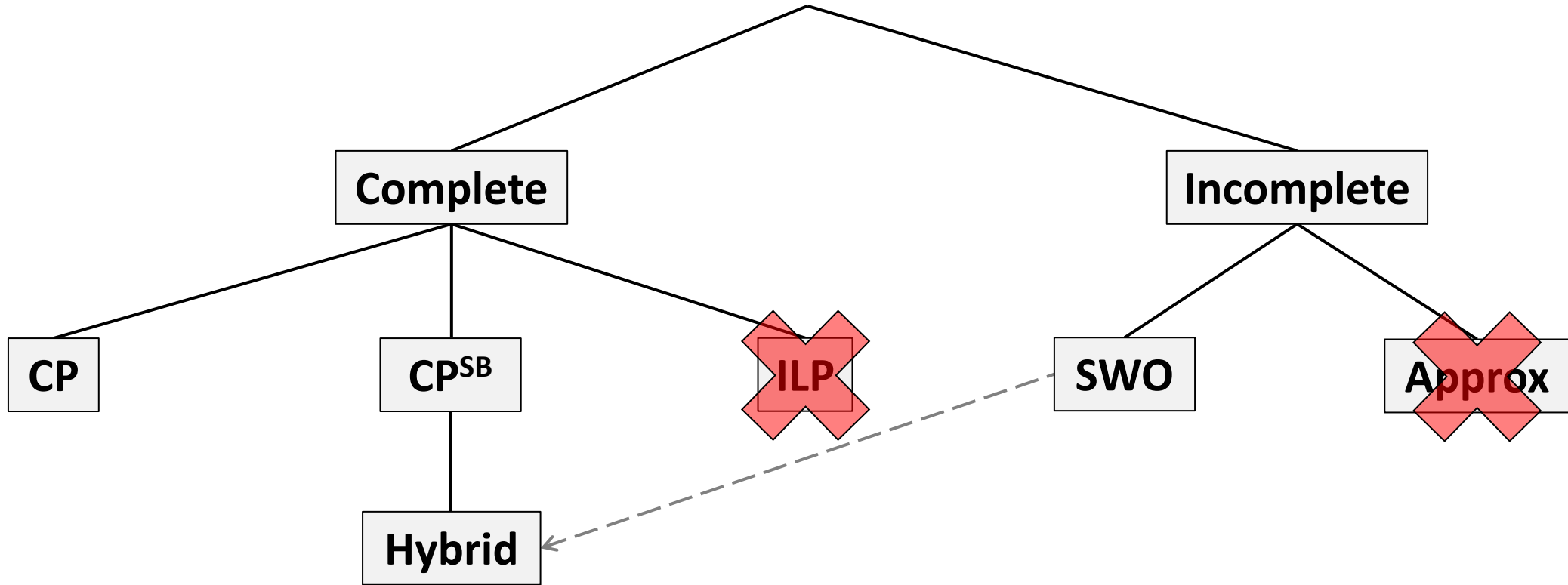
# Solving Approaches



# Solving Approaches



# Solving Approaches



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I. Problem Description

II. Modeling

III. Solving Approaches

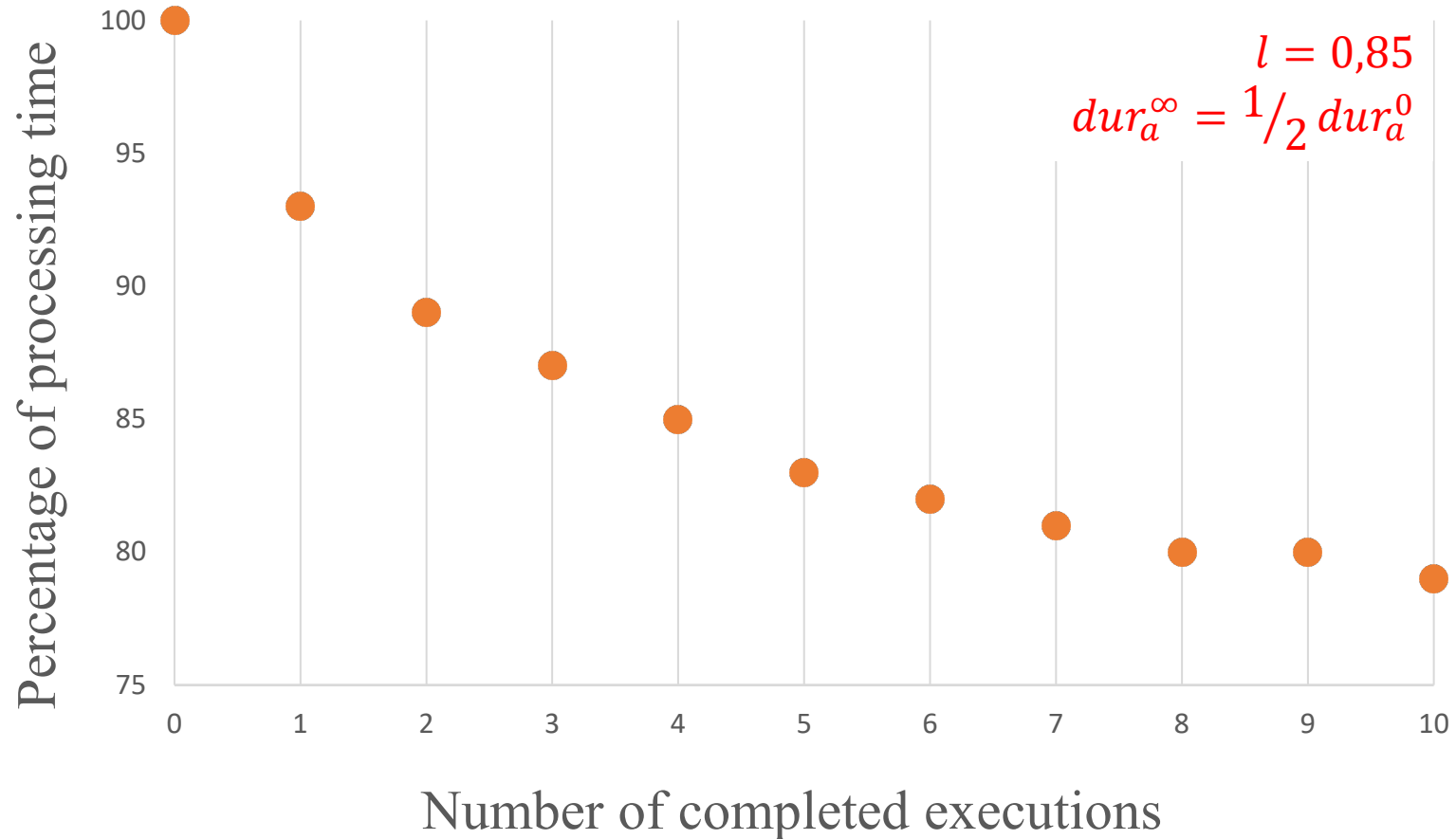
IV. Experimentation

V. Conclusion



# Learning Curve

$$\forall n \in \mathbb{N}, \quad dur_a(n) = dur_a^\infty + [(dur_a^0 - dur_a^\infty) * (n + 1)^{\log_2 l}]$$



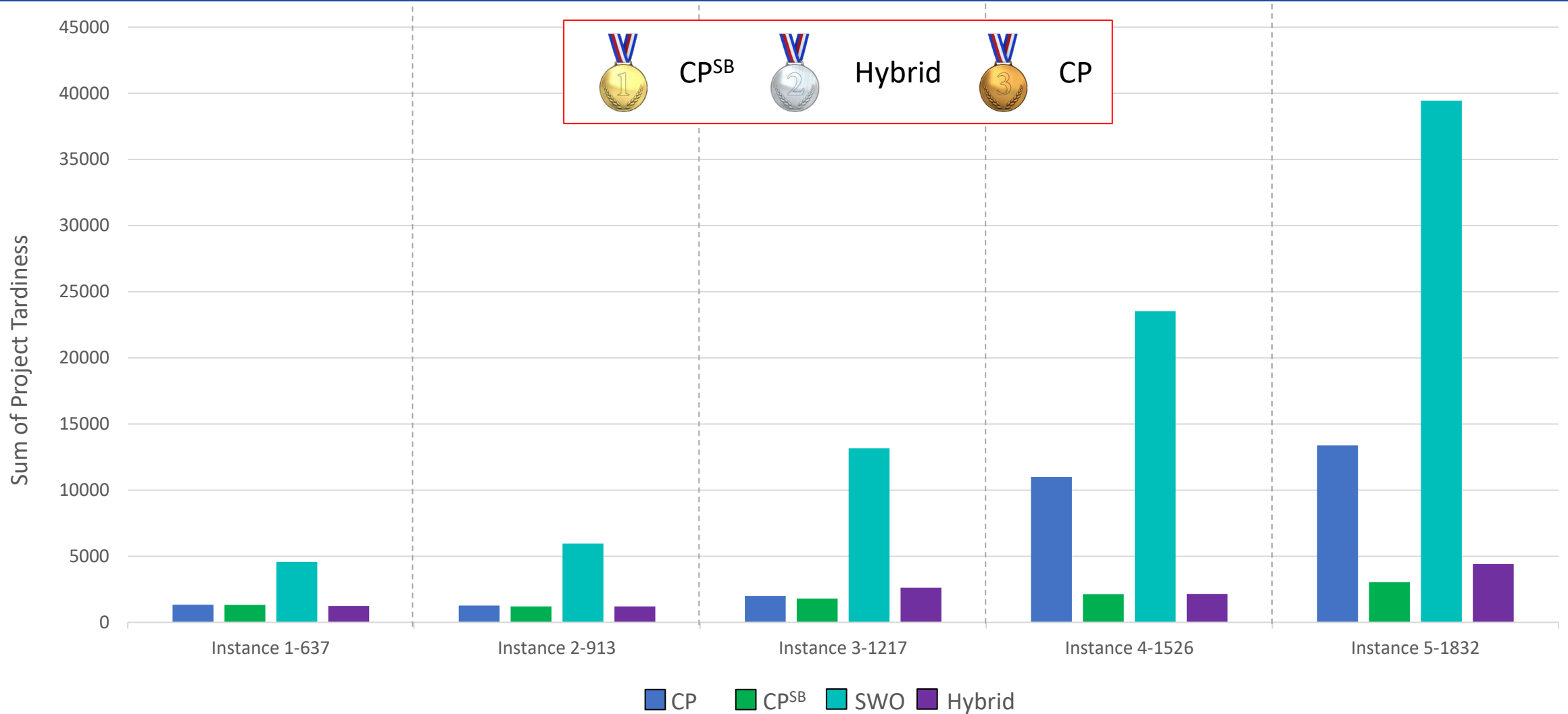
# Industrial Benchmarks

Instances		$ C $	$ A_c $	$ A_c^{Comp} $	$ I_c $	$ R $	$ capa_r $	$l$
Satellite	Original	3	$\leq 30$	$\leq 3$	$\leq 5$	40	$\leq 16$	[0.05,0.95]
	Extended	6			$\leq 14$			0.85

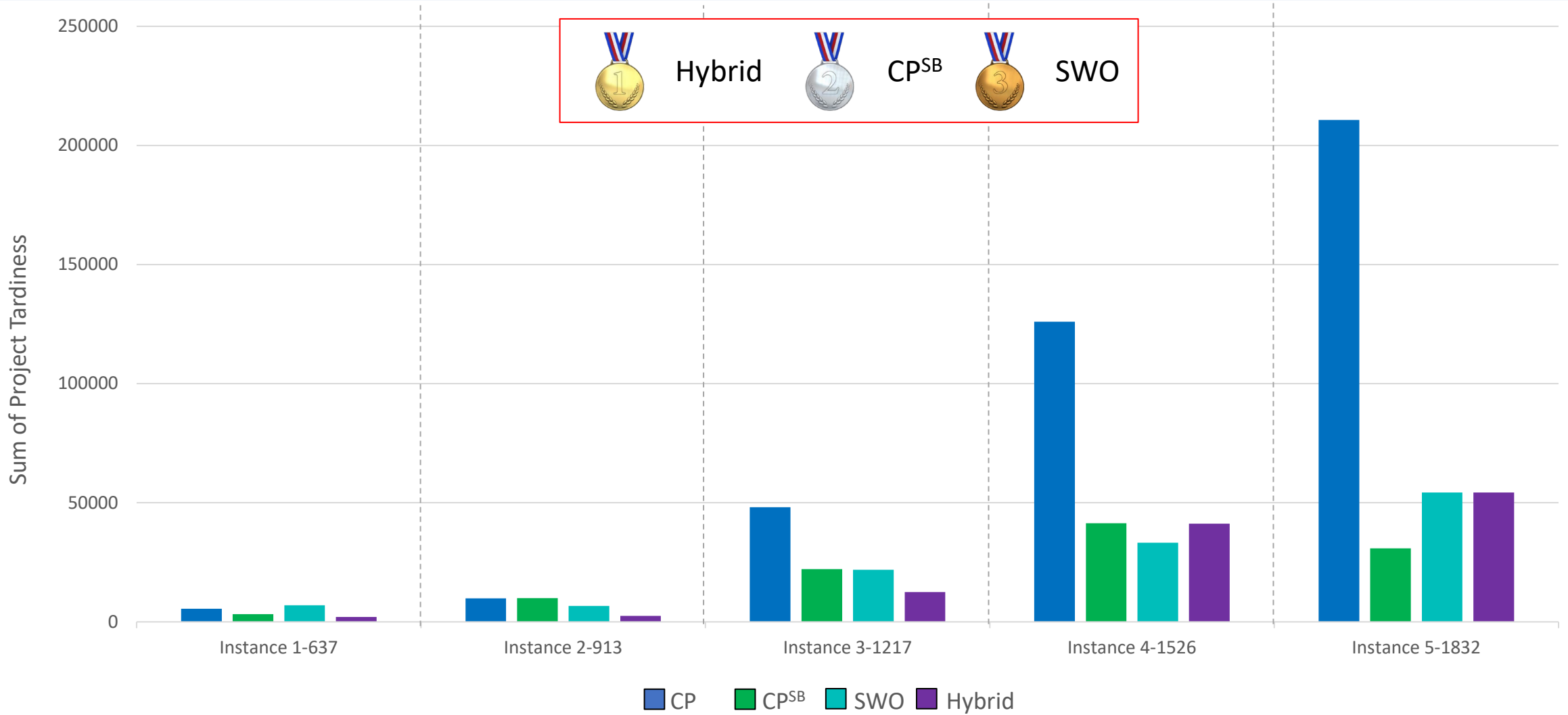
Test environment:

- IBM CP Optimizer 22.1.1 / Julia 1.10.1
- Timeout: 2h / 15s

# Industrial Benchmarks (2h)



# Industrial Benchmarks (15s)



# PSP-based Benchmarks

1 class = 1 RCPSP from PSPLIB

- Several classes
- Multiplicity projects
- Random due dates
- Random learning rate
- Resource capacities = sum of PSPLIB resource capacities

Instances		$ C $	$ A_c $	$ A_c^{Comp} $	$ I_c $	$ R $	$ capa_r $	$l$
PSP-based	Small	$[[2,4]]$	$\{30,60\}$	0	$[[5,10]]$	4	$\sum_{i_{LIB}} capa_r^{i_{LIB}}$	$[0.45,0.95]$
	Large	$[[5,7]]$	$\{60,90,120\}$					

# PSP-based Benchmarks

- 50 instances
- Dataset available online

Timeout	Dataset	 CP	 CP <sup>SB</sup>	SWO	 Hybrid
2h	Small	21	21	0	21
	Large	17	16	0	16
15s	Small	22	16	0	19
	Large	15	4	6	3

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# V. Conclusion

---

- ✓ Learning effect is important in HM-RCPS
- ✓ CP-based approaches showed the best performance in tests



# V. Conclusion

---

- ✓ Learning effect is important in HM-RCPSP
- ✓ CP-based approaches showed the best performance in tests
- Explore another potential metaheuristic

# V. Conclusion

---

- ✓ Learning effect is important in HM-RCPSp
- ✓ CP-based approaches showed the best performance in tests
  
- Explore another potential metaheuristic
- Dominance breaking variations

# V. Conclusion

---

- ✓ Learning effect is important in HM-RCPSP
  - ✓ CP-based approaches showed the best performance in tests
- 
- Explore another potential metaheuristic
  - Dominance breaking variations
  - Learning effect on similar activities between classes

# V. Conclusion

---

- ✓ Learning effect is important in HM-RCPSP
  - ✓ CP-based approaches showed the best performance in tests
- 
- Explore another potential metaheuristic
  - Dominance breaking variations
  - Learning effect on similar activities between classes
  - Uncertainty learning duration

**Thank you for listening!**