Minimizing Working-Group Conflicts in Conference Session Scheduling Through Maximum Satisfiability

Sami Cherif, Heythem Sattoutah, Chu-Min Li
Corinne Lucet, Laure Brisoux-Devendeville

CP 2024

Plan

Preliminaries

literal

A literal is a variable x, or its negation $\overline{x}$

Clause

A clause is a disjunction of literals $(l_1 \vee l_2 \vee \cdots \vee l_m)$

Conjunctive Normal Form (CNF)

A CNF formula is a conjunction of clauses $C_1 \wedge C_2 \wedge \cdots \wedge C_n$

$\phi = (x_1 \vee x_2) \wedge (\overline{x_1} \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})$
The formula ϕ is satisfied by $\alpha = \{x_1, \overline{x_2}, x_3\}$

Maximum Satisfiability (Max-SAT)

Definition

Given a CNF formula ϕ, Max-SAT consists in determining the maximum (resp. minimum) number of clauses that can be satisfied (resp. falsified) by an assignment of the variables :

$$opt(\phi) = \min_{\alpha} \; cost_{\alpha}(\phi)$$

where $cost_{\alpha}(\phi)$ denotes the number of clauses falsified by α.

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where $cost_{\alpha}(\phi)$ denotes the number of clauses falsified by α.

$\phi = (x_1 \vee x_2) \wedge (\overline{x_1} \vee x_3) \wedge (\overline{x_2} \vee \overline{x_3})$

- For $\alpha_1 = \{x_1, x_2, \overline{x_3}\}$, $cost_{\alpha}(\phi) = 1$
- For $\alpha_2 = \{x_1, \overline{x_2}, x_3\}$, $cost_{\alpha}(\phi) = opt(\phi) = 0$

Partial Max-SAT

Partial CNF Formula

A partial CNF formula is a bipartite set of clauses $\phi = S \cup H$ where H denotes the set of hard clauses and S the set of soft clauses

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Partial Max-SAT

Given a partial CNF formula $\phi = H \cup S$, Partial Max-SAT consists in determining the minimum number of soft clauses that can be falsified by an assignment while satisfying all the hard clause in ϕ:

$$opt(\phi) = \min_{\alpha \text{ s.t H satisfied}} cost_{\alpha}(S)$$

Partial Max-SAT

$\phi = H \cup S$ with $H = \{(x_1 \vee x_2)\}$ and $S = \{(\overline{x_1} \vee x_3), (\overline{x_2} \vee \overline{x_3})\}$

- $\alpha_1 = \{\overline{x_1}, \overline{x_2}, x_3\}$ is not a feasible solution because the hard clause is not satisfied
- $\alpha_2 = \{x_1, x_2, \overline{x_3}\}, cost_{\alpha_2}(S) = 1$ is a feasible solution
- $\alpha_3 = \{\overline{x_1}, x_2, \overline{x_3}\}, cost_{\alpha_3}(S) = 0$ is an optimal solution !

Partial Max-SAT

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- $\alpha_2 = \{x_1, x_2, \overline{x_3}\}$, $cost_{\alpha_2}(S) = 1$ is a feasible solution
- $\alpha_3 = \{\overline{x_1}, x_2, \overline{x_3}\}$, $cost_{\alpha_3}(S) = 0$ is an optimal solution !

Pseudo-Boolean Constraints

$(\sum_{i=1}^{h} a_i * l_i) \circ k$ where $a_i, k \in \mathbb{N}$ and $\circ \in \{\leq, =, \geq\}$.

Plan

ROADEF Conference

ROADEF
SOCIÉTÉ FRANÇAISE DE RECHERCHE OPÉRATIONNELLE
ET D'AIDE À LA DÉCISION

- Largest french-speaking conference dedicated to combinatorial optimisation and operations research.
- Annual event, with around 600 participants
- Comprises plenary sessions, **multiple parallel sessions**, industrial feedback sessions, ROADEF general assembly, ...
- Aims to train young researchers and to promote exchanges and collaborations with industrial partners

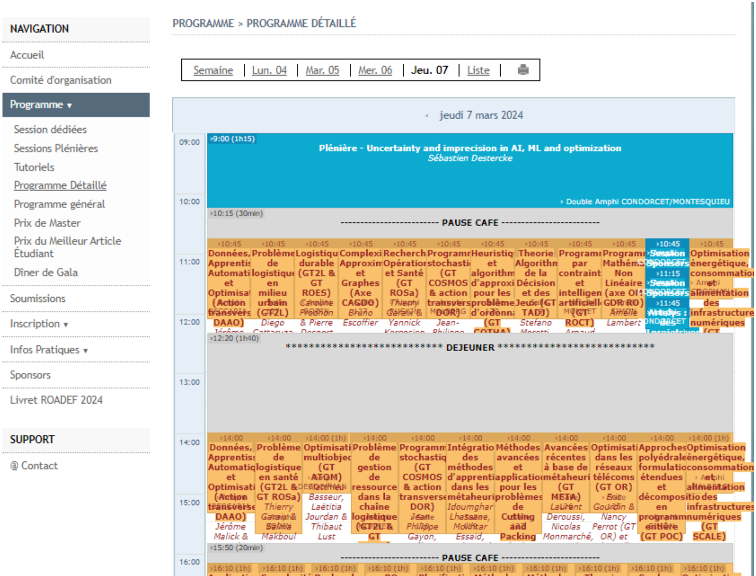
ROADEF Scheduling

Mercredi 6 mars		
9h – 10h15	Plénière Nadia Brauner	Amphis
10h15 – 10h45	Pause café	Hall
10h45 – 12h05	Sessions Parallèles #5 (4)	
12h – 14h	Pause déjeuner	Restaurant Universitaire
14h – 14h30	AG GdR RO	Amphi
14h45 – 15h45	Tutoriels I	Petit Amphi
14h45 – 15h45	Tutoriels II	Petit Amphi
14h45 – 15h45	Prix Master (3)	
16h05 – 16h45	Prix Master (2)	
14h45 – 15h45	Sponsors	Petit Amphi
15h45 – 16h05	Pause café	Hall
17h – 18h30	AG de la ROADEF	Amphis

Mardi 5 mars		
8h – 8h45	Accueil	
8h45 – 9h15	Session d'ouverture	Amphis
9h15 – 10h30	Plénière Carola Doer	Amphis
10h30 – 11h	Pause café	Hall
11h – 12h20	Sessions Parallèles #1 (4)	
12h – 14h	Pause déjeuner	Restaurant Universitaire
14h – 16h	Prix Etudiant I (4)	
14h – 16h	Sessions Parallèles #2 (6)	
16h30 – 18h30	Sessions Parallèles #3 (3-6)	
16h – 16h30	Pause café	Hall
16h30 – 18h	Prix Etudiant II (3)	
~~17h10 – 18h30~~	~~Sessions Parallèles #4 (4)~~	

Jeudi 7 mars		
9h – 10h15	Plénière Sébastien	Amphis
10h15 – 10h45	Pause café	Hall
10h45 – 12h20	Sessions Parallèles #6 (4)	
12h – 14h	Pause déjeuner	Restaurant Universitaire
14h – 15h40	Sessions Parallèles #7 (5)	
15h50 – 16h10	Pause café	Hall
16h10 – 17h30	Sessions Parallèles #8 (4)	
18h	Départ repas de Gala	

ROADEF Scheduling



Working Groups & Sessions

Working Groups

Group of researchers / Research field

Sessions

- A session is organised in one or several parallel session slots
- A session is generally organised by one or several working groups

Problem

Overlaps prevent researchers from participating in their group sessions.

Objective

Avoid overlaps (conflicts) between working groups in available time slots, in order to allow optimal participation of researchers in their sessions.

Manuel Solution by the organising committee:

				Mardi			Mercredi	Jeudi			
fr	CASID	NB papiers		1 (4)	2(6)	3 (3-6)	5 (4)	6(4)	7(5)	8(4)	
Sessions Parallv®les	NB Sessions //			11	11	11	11	11	11	11	
Theorie Algorithmique de la Dv©cision et des Jeux (GT TADJ)	1453676	14		4		3		4		3	14
Donnv©es, Apprentissage Automatique et Optimisation (Action transverse DAAO)	1453676	23			6	4	4	4	5		23
Prix du Meilleur Article √âtudiant	1453676										0
Prix du Master	1453676										0
Approches exactes basv©es sur l'apprentissage pour l'optimisation des systv®mes logistiques	1453676	2									0
Recherche Opv©rationnelle et Santv© (GT ROSA)	201292	12		3		3		3		3	12
Mv©thodes avancv©es et applications pour les problv®mes de Cutting and Packing	148991	9					4		5		9
Intv©gration des mv©thodes d'apprentissage dans les mv©taheuristiques	190261	9			6				3		9
RO et v©thique (GT ROET)	120058	2									0
RO Environnement & Sociv©tv© (GT ROES) + GT ROET	50035	4	6			3				3	6
Optimisation bi-niveaux et applications	1453676	6			6						6

Plan

Formalisation

1. **Sessions (S)**
2. **Working Groups (G)**
3. **Available Slots (C)**
4. **Amount of authorized papers per session (L)**

Formalisation

6. n: **maximal number of parallel sessions**
7. $np(s)$: **number of papers accepted in session** $s \in S$
8. $npMax(c)$: **maximal number of papers per slot** $c \in C$
9. $WG(s) \subseteq G$: **Working Groups for session** $s \in S$

Case study: ROADEF 2024

- 25^{th} edition organised in Amiens by the MIS lab
- 40 sessions, 20 groups, 7 slots.

Constraints

- 20 minutes per presentation
- Each parallel session should last between 1 and 2 hours
- A maximum of 15 sessions in parallel

Case study: ROADEF 2024

- $S = \{1, \ldots, 40\}$: Sessions
- $G = \{1, \ldots, 20\}$: Groups
- $C = \{1, \ldots, 7\}$: Slots
- $L = \{3, 4, 5, 6\}$: amount of papers per session
- $n \leq 15$: maximal number of parallel sessions (per slot)

307 papers accepted in the conference distributed over the sessions.

Case study: ROADEF 2024

Session s	Label	np(s)	Groups	WG(s)
1	Theorie Algorithmique de la Décision et des Jeux	14	TADJ	{1}
2	Données, Apprentissage Automatique et Optimisation	23	DAAO	{2}
.....	....	....	......	......
39	Problème de logistique en milieu urbain	6	GT2L, P2LS	{8,12}
40	Programmation par contraintes et intelligence artificielle	8	ROCT, META	{7,15}

Slot (c)	1	2	3	4	5	6	7
npMax(c)	4	6	6	4	4	5	3

Case study: ROADEF 2024

Session s	Label	np(s)	Groups	WG(s)
1	Theorie Algorithmique de la Décision et des Jeux	14	TADJ	{1}
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Slot (c)	1	2	3	4	5	6	7
npMax(c)	4	6	6	4	4	5	3

Specific Constraints

Session 34 must be programmed in the last day (slots 5, 6, 7)

Variables

- **Assignment Variables** $x_{(s,c,l)}$

  True if session s is allocated to slot c with l papers.

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- **Assignment Variables** $x_{(s,c,l)}$

  True if session s is allocated to slot c with l papers.

- **Conflict Variables** $y_{(s_1,s_2,c,g)}$

  True if there is a conflict for group g between sessions s_1 and s_2 in slot c.

Soft Clauses

Targeted Objective

Minimise the number of conflicts, i.e. the number of satisfied y variables

$$\phi_{soft} = \bigwedge_{\substack{(s_1,s_2,c,g)\in S\times S\times C\times G \\ s1<s2 \\ g\in W(s_1)\cap W(s_2)}} \overline{y_{(s_1,s_2,c,g)}}$$

Hard Constraints

Constraints 1 & 2:

- At most one quantity of papers for each (session,slot) pair:

$$\sum_{l \in L} x_{(s,c,l)} \leq 1 \qquad \forall (s,c) \in S \times C \tag{1}$$

Hard Constraints

Constraints 1 & 2:

- At most one quantity of papers for each (session,slot) pair:

$$\sum_{l \in L} x_{(s,c,l)} \leq 1 \qquad \forall (s,c) \in S \times C \tag{1}$$

- Cover all the papers in a session:

$$\sum_{\substack{c \in C \\ l \in L}} x_{(s,c,l)} \times l = np(s) \qquad \forall s \in S \tag{2}$$

Hard Constraints

Constraints 3 & 4 & 5 :

Enforce the maximal number of papers per slot:

$$\bigwedge_{\substack{l \in L \\ l > npMax(c)}} \overline{x(s,c,l)} \qquad \forall (s,c) \in S \times C \tag{3}$$

Hard Constraints

Constraints 3 & 4 & 5 :

Enforce the maximal number of papers per slot:

$$\bigwedge_{\substack{l \in L \\ l > npMax(c)}} \overline{x(s,c,l)} \qquad \forall (s,c) \in S \times C \tag{3}$$

Enforce the maximal number of parallel sessions:

$$\sum_{\substack{s \in S \\ l \in L}} x_{(s,c,l)} \leq n \qquad \forall c \in C \tag{4}$$

Hard Constraints

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Enforce the maximal number of papers per slot:

$$\bigwedge_{\substack{l \in L \\ l > npMax(c)}} \overline{x(s,c,l)} \qquad \forall (s,c) \in S \times C \tag{3}$$

Enforce the maximal number of parallel sessions:

$$\sum_{\substack{s \in S \\ l \in L}} x_{(s,c,l)} \leq n \qquad \forall c \in C \tag{4}$$

Session 34 must be programmed in the last day of the conference:

$$\bigwedge_{\substack{c \in C \setminus \{5,6,7\} \\ l \in L}} \overline{x_{(34,c,l)}} \tag{5}$$

Hard Constraints - Conflict Management

Two sessions associated to the same group and assigned to the same slot generate a conflict:

$$(\sum_{l \in L} x_{(s_1,c,l)} \geq 1 \bigwedge \sum_{l \in L} x_{(s_2,c,l)} \geq 1) \implies y_{(s_1,s_2,c,g)}$$

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$$(\bigwedge_{l \in L} \overline{x_{(s_1,c,l)}}) \bigvee (\bigwedge_{l \in L} \overline{x_{(s_2,c,l)}}) \bigvee y_{(s_1,s_2,c,g)} \quad (6)$$

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Introduction of new variables

- $z_{(s,c)}$ is true if the session s is not allocated to slot c.

Hard Constraints - Conflict Management

Constraints 6 & 7 :

- Enforce the semantic meaning of the z variables:

$$z_{(s,c)} \Leftrightarrow \bigwedge_{l \in L} \overline{x_{(s,c,l)}} \qquad \forall (s,c) \in S \times C \tag{7}$$

Hard Constraints - Conflict Management

Constraints 6 & 7 :

- Enforce the semantic meaning of the z variables:

$$z_{(s,c)} \Leftrightarrow \bigwedge_{l \in L} \overline{x_{(s,c,l)}} \qquad \forall (s,c) \in S \times C \tag{7}$$

- Two sessions associated to the same group and assigned to the same slot generate a conflict:

$$z_{(s_1,c)} \vee z_{(s_2,c)} \vee y_{(s_1,s_2,c,g)} \tag{6'}$$

$$\forall (s_1, s_2, c, g) \in S \times S \times C \times G \text{ s.t } s1 < s2 \text{ and } g \in WG(s_1) \cap WG(s_2)$$

Hard Constraints - Reformulations

Reformulation of Constraints 4 & 5 :

- Enforce the maximal number of parallel sessions:

$$\sum_{s \in S} \overline{z_{(s,c)}} \leq n \qquad \forall c \in C \tag{4'}$$

Hard Constraints - Reformulations

Reformulation of Constraints 4 & 5 :

- Enforce the maximal number of parallel sessions:

$$\sum_{s \in S} \overline{z_{(s,c)}} \leq n \qquad \forall c \in C \tag{4'}$$

- Session 34 must be programmed in the last day of the conference:

$$\bigwedge_{c \in C \setminus \{5,6,7\}} z_{(34,c)} \tag{5'}$$

Plan

Experimental Protocol

- Model(s) implemented in PySAT[1] with sorting-network encoding for Pseudo-Boolean constraints[2].
- 84 generated instances extracted from 2021-2024 data, with 12 UNSAT
- Timeout set to 3600s per instance.
- Solvers : RC2, OpenWBO, MaxCDCL and MaxHS
- Comparison with CP-SAT of OR-Tools (CP Approach)

[1] https://pysathq.github.io/

[2] $O(m \times \log^2(m))$ where m is the sum of weights

Results : Model vs Handmade Solutions

Year	2024	2023	2022	2021
Sessions	40	47	42	27
Groups	20	24	24	17
Slots	7	7	8	11
Papers	307	358	311	182
Conflicts	4 (6)	9 (35)	29 (38)	0 (3)
Parallel Sessions	10 (11)	13 (14)	11 (11)	5 (5)

Results : Formulations

		Without Z					With Z				
Year	n	RC2	WBO	Max-CDCL	Max-HS	CP-SAT	RC2	WBO	Max-CDCL	Max-HS	CP-SAT
2024	15	**3.428**	7.811	5.358	16.086	26.800	1.259	1.180	**0.989**	17.933	7.683
	14	**3.054**	12.095	5.716	46.308	26.291	1.300	1.130	**0.818**	14.633	6.677
	13	**3.288**	6.989	5.591	46.206	20.622	1.199	1.404	**0.992**	18.585	7.713
	12	**3.540**	6.036	5.147	95.241	22.691	1.315	1.742	**0.805**	20.607	10.056
	11	**3.652**	12.384	8.746	153.335	22.594	1.287	2.472	**1.100**	71.655	10.390
	10	**9.902**	1398.111	511.043	869.119	88.831	**9.707**	341.225	150.416	271.440	35.460
2023	17	197.515	**119.674**	T (-)	1187.325	T (-)	**9.687**	17.769	T (9)	357.906	T (9)
	16	355.627	**71.472**	T (-)	755.975	T (9)	**12.602**	20.701	3058.418	395.716	T (9)
	15	139.625	**99.332**	T (-)	1106.281	T (9)	**23.467**	41.042	T (9)	408.854	T (9)
	14	107.202	**102.947**	T (-)	1036.948	T (9)	**14.011**	22.683	T (9)	413.160	T (9)
	13	250.308	**199.549**	T (-)	853.876	T (9)	**8.006**	14.962	3136.008	439.812	T (9)
	12	T [10]	T (-)	T (-)	T (28)[8]	T (-)	T (-)[10]	T (-)	T (-)	T (23)[8]	T (10)
2022	16	T (-)[24]	T (127)	**T (29)**	385.625 (46)[18]	**T (29)**	T (-)[25]	T (146)	**T (29)**	496.061 (42)[22]	**T (29)**
	15	T (-)[24]	T (161)	**T (29)**	380.048 (37)[26]	**T (29)**	T (-)[24]	T (134)	**T (29)**	322.627 (51)[18]	**T (29)**
	14	T (-)[25]	T (196)	**T (29)**	T (37)[26]	**T (29)**	T (-)[27]	T (132)	**T (29)**	473.504 (38)[23]	**T (29)**
	13	T (-)[24]	T (222)	**T (29)**	392.299 (53)[18]	**T (29)**	T (-)[27]	T (146)	**T (29)**	348.256 (56)[17]	**T (29)**
	12	T (-)[24]	T (324)	**T (29)**	856.923 (50)[24]	**T (29)**	T (-)[26]	T (135)	**T (29)**	980.692 (30)[25]	**T (29)**
	11	T (-)[25]	T (543)	**T (29)**	T (40)[27]	**T (29)**	T (-)[27]	T (436)	**T (29)**	713.466 (56)[18]	**T (29)**
2021	10	0.670	1.397	**0.503**	0.653	1.356	0.656	0.748	0.179	**0.132**	1.307
	9	1.210	1.380	**0.276**	0.685	1.324	0.309	0.812	0.170	**0.155**	1.316
	8	1.232	1.352	**0.437**	0.682	1.327	0.654	0.775	**0.278**	0.454	1.272
	7	1.189	1.364	**0.262**	0.619	1.333	0.649	0.791	**0.256**	0.448	1.276
	6	0.646	1.384	**0.414**	0.695	1.307	0.350	0.776	**0.281**	0.470	1.274
	5	1.142	1.724	**0.425**	0.696	1.347	0.661	0.885	**0.279**	0.459	1.280

Results : Encodings

		SortN				CardN+PBbest			
Année	**n**	**RC2**	**WBO**	**Max-CDCL**	**Max-HS**	**RC2**	**WBO**	**Max-CDCL**	**Max-HS**
2024	15	1.259	1.180	**0.989**	17.933	0.767	**0.300**	0.467	9.925
	14	1.300	1.130	**0.818**	14.633	0.716	0.301	**0.290**	11.236
	13	1.199	1.404	**0.992**	18.585	0.770	**0.279**	0.353	12.476
	12	1.315	1.742	**0.805**	20.607	0.718	**0.309**	0.313	5.972
	11	1.287	2.472	**1.100**	71.655	0.707	0.392	0.395	26.609
	10	**9.707**	341.225	150.416	271.440	**1.544**	411.766	213.257	500.964
2023	17	**9.687**	17.769	T (9)	357.906	10.370	**6.136**	3371.304	353.515
	16	**12.602**	20.701	3058.418	395.716	12.145	**4.792**	2866.853	426.965
	15	**23.467**	41.042	T (9)	408.854	15.247	***6.696***	2686.484	405.923
	14	**14.011**	22.683	T (9)	413.160	19.751	**6.706**	T (9)	417.693
	13	**8.006**	14.962	3136.008	439.812	8.733	**4.162**	T (9)	438.877
	12	T (-)[10]	T (-)	T (-)	T (23)[8]	T [10]	T (-)	T (-)	T (30)[9]
2022	16	T (-)[25]	T (146)	**T (29)**	496.061 (42)[22]	T (-)[22]	T (83)	**T (29)**	1024.785 (40)[25]
	15	T (-)[24]	T (134)	**T (29)**	322.627 (51)[18]	T (-)[24]	T (58)	**T (29)**	1059.200 (33)[25]
	14	T (-)[27]	T (132)	**T (29)**	473.504 (38)[23]	T (-)[24]	T (47)	**T (29)**	**448.613 (29)[23]**
	13	T (-)[27]	T (146)	**T (29)**	348.256 (56)[17]	T (-)[25]	T (50)	**T (29)**	330.284 (59)[18]
	12	T (-)[26]	T (135)	**T (29)**	980.692 (30)[25]	T (-)[24]	T (71)	**T (29)**	328.658 (59)[18]
	11	T (-)[27]	T (436)	**T (29)**	713.466 (56)[18]	T (-)[24]	T (287)	**T (29)**	650.517 (48)[18]
2021	10	0.656	0.748	0.179	**0.132**	0.178	0.139	0.050	**0.027**
	9	0.309	0.812	0.170	**0.155**	0.133	0.136	0.038	**0.026**
	8	0.654	0.775	**0.278**	0.454	0.132	0.148	0.039	**0.030**
	7	0.649	0.791	**0.256**	0.448	0.126	0.132	0.035	**0.030**
	6	0.350	0.776	**0.281**	0.470	0.135	0.146	0.035	**0.031**
	5	0.661	0.885	**0.279**	0.459	0.142	0.200	0.035	**0.027**

Results

- Timeout for $n = 9$ (2024), $n = 11$ (2023), $n = 10$ (2022), $n = 4$ (2021) by all Max-SAT solvers
- These n values correspond to **UNSAT** instances
- For example, in 2024

$$\sum_{c \in C} n \times npMax(c) = 288 < 307$$

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Plan

Conclusion

- Solving a conference scheduling optimisation problem through Max-SAT
- Simplifying the planning task, streamlining logistics and optimizing resources for subsequent editions of the conference.
- A Set of hard UNSAT instances for Max-SAT

- Improve our model and enlarge our benchmark to other editions/conferences
- Take into consideration other constraints

Minimizing Working-Group Conflicts in Conference Session Scheduling Through Maximum Satisfiability

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CP 2024