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Structure-Guided Local Improvement for Maximum Satisfiability

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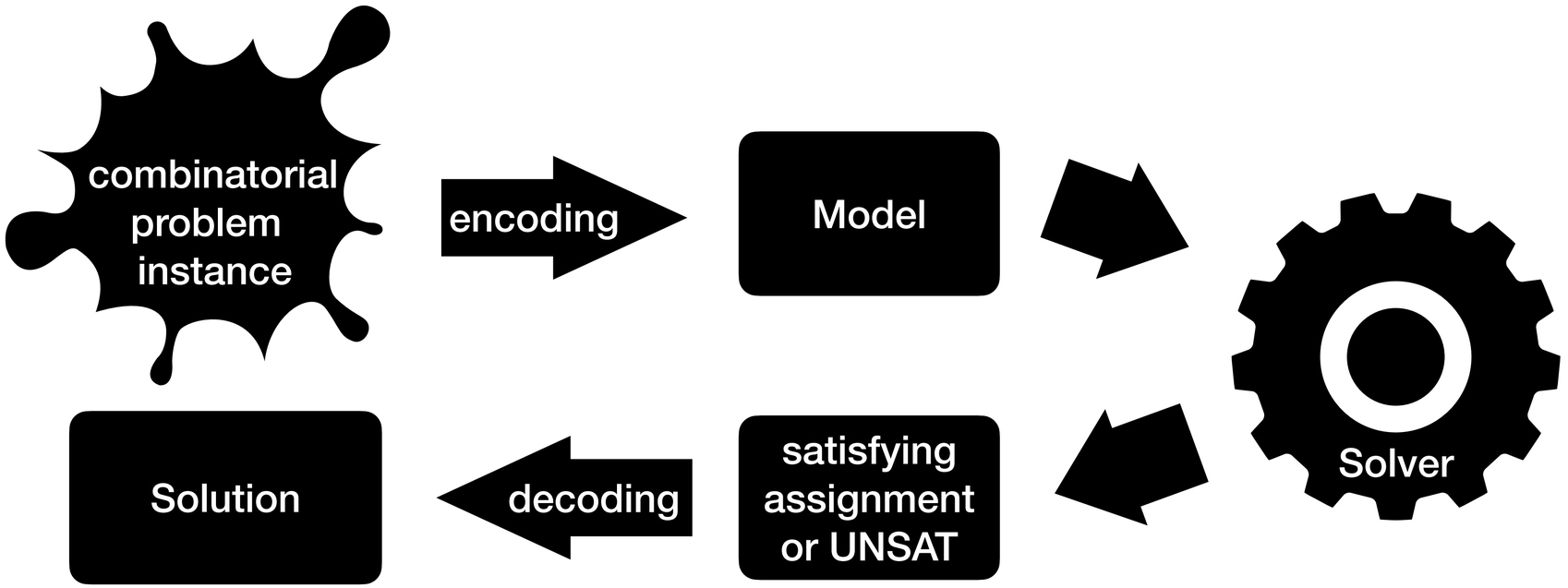
One-Shot Encodings



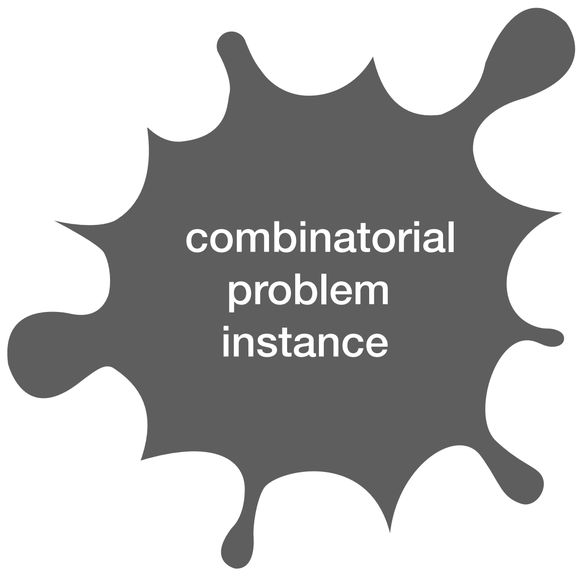
Limits

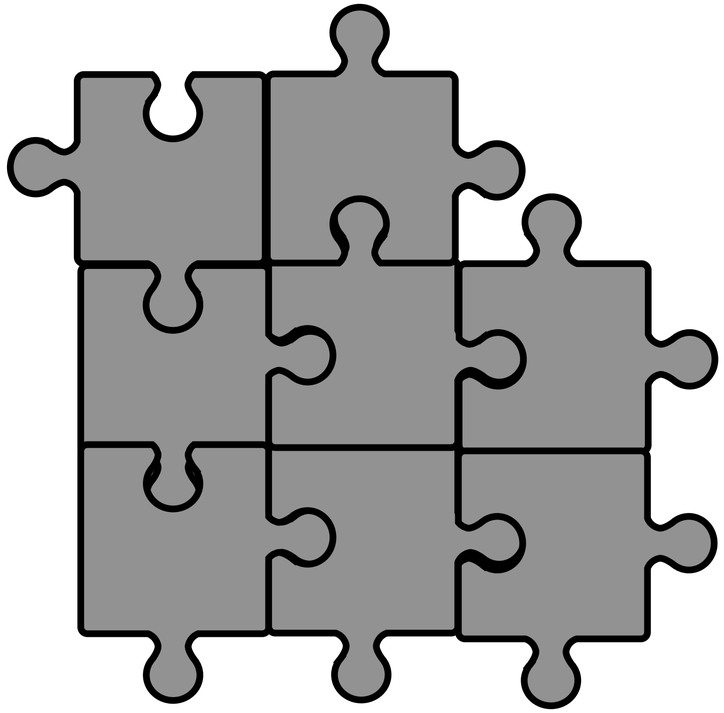
- Encoding size if often too large to do a one-shot encoding
- Even if one-shot encoding is possible, solvers sometimes perform better on a smaller instance

SLIM (SAT-based Local Improvement)

SLIM

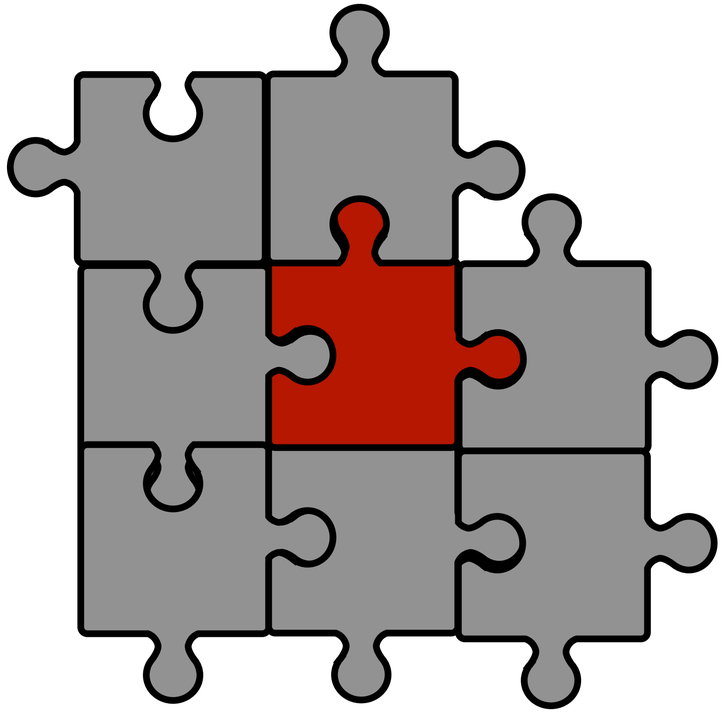
Compute a heuristic solution





SLIM

select a local part of the instance

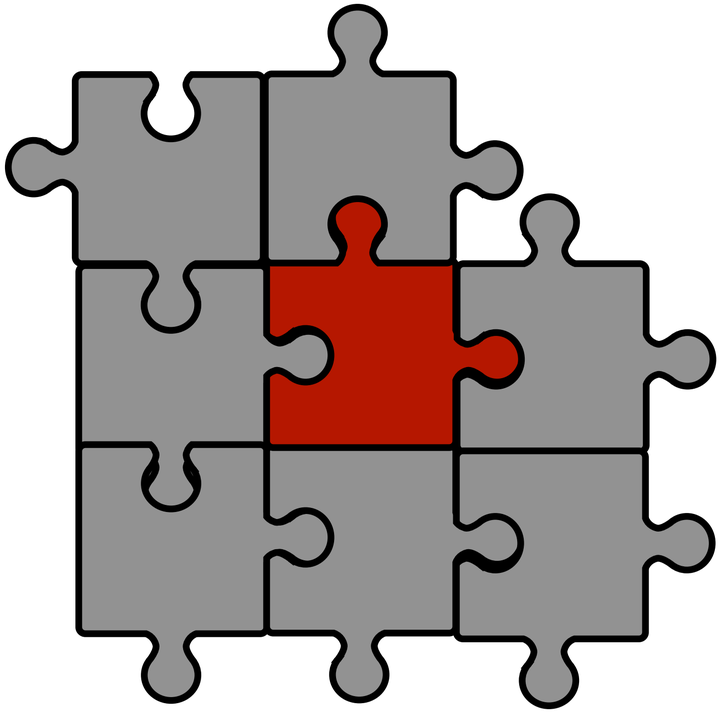


local selection strategy to
decide where and how large to
select (*budget*)

SLIM

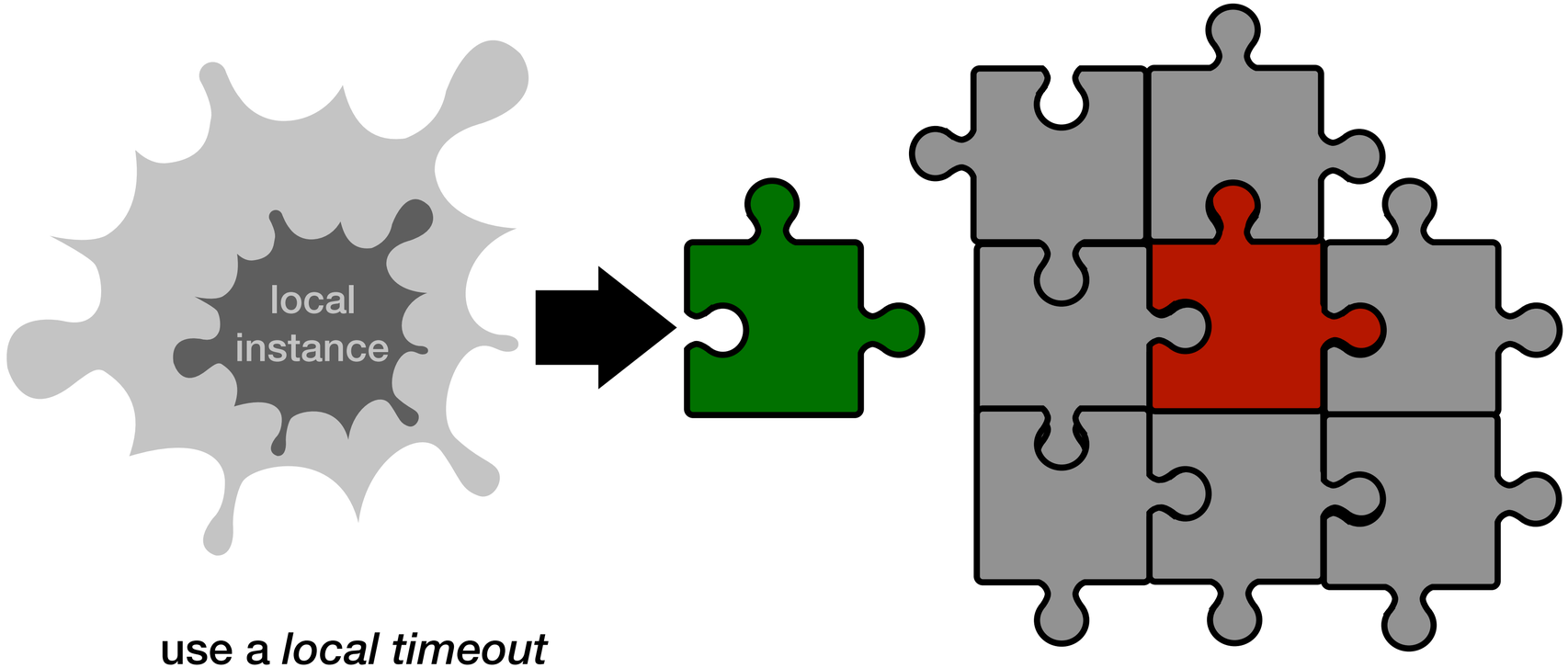
create the corresponding local instance

more constrained,
to ensure *replacement consistency*



SLIM

solve the local instance with a solver



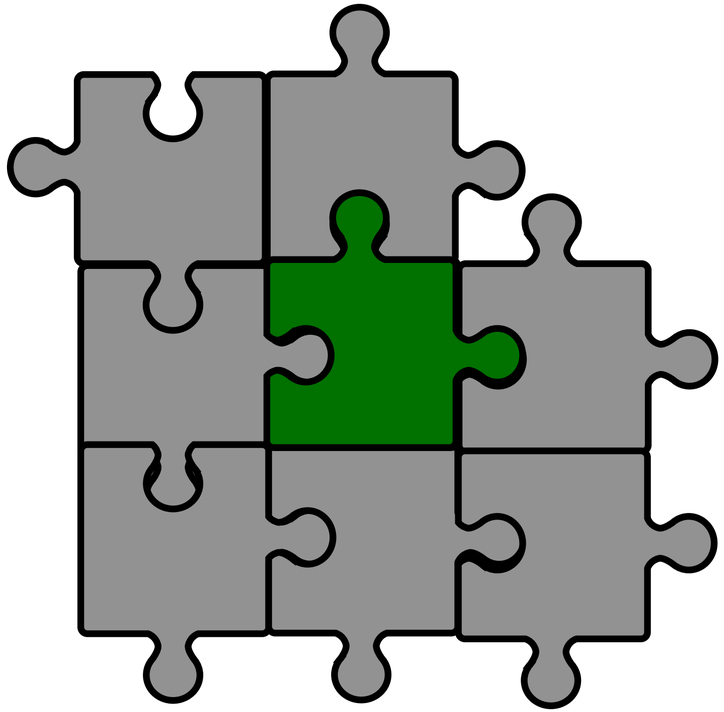
use a *local timeout*
solve not necessarily optimally or until UNSAT

SLIM

fit the local solution into the global solution

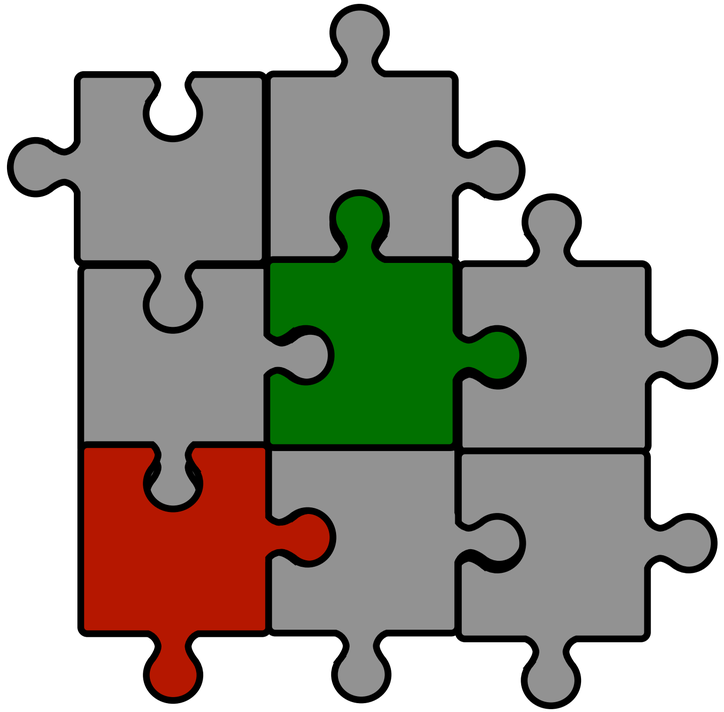
works due to the additional constraints

replace even if new solution is not better



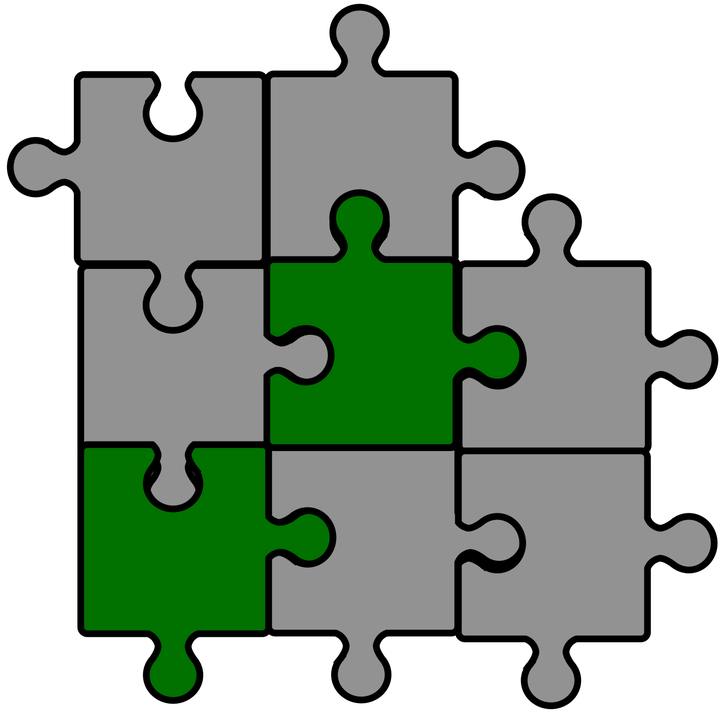
SLIM

repeat



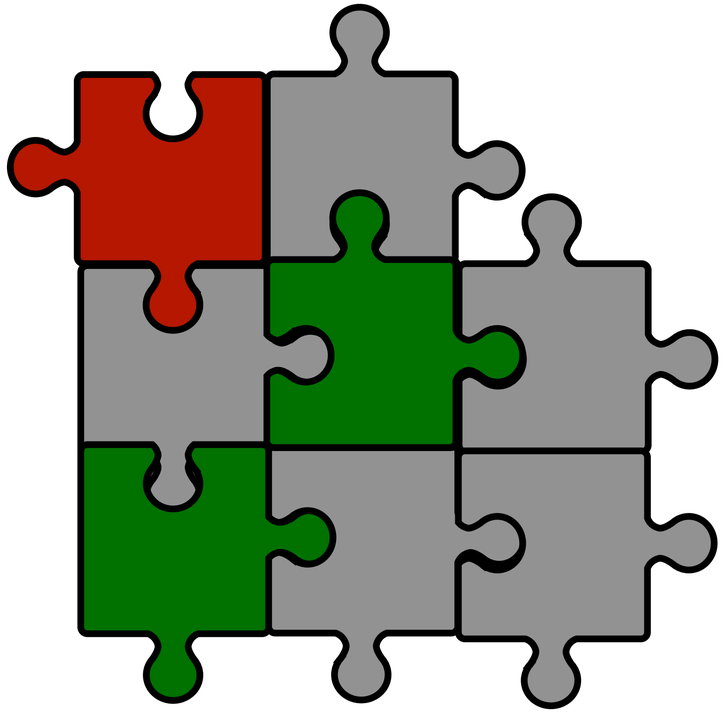
SLIM

repeat



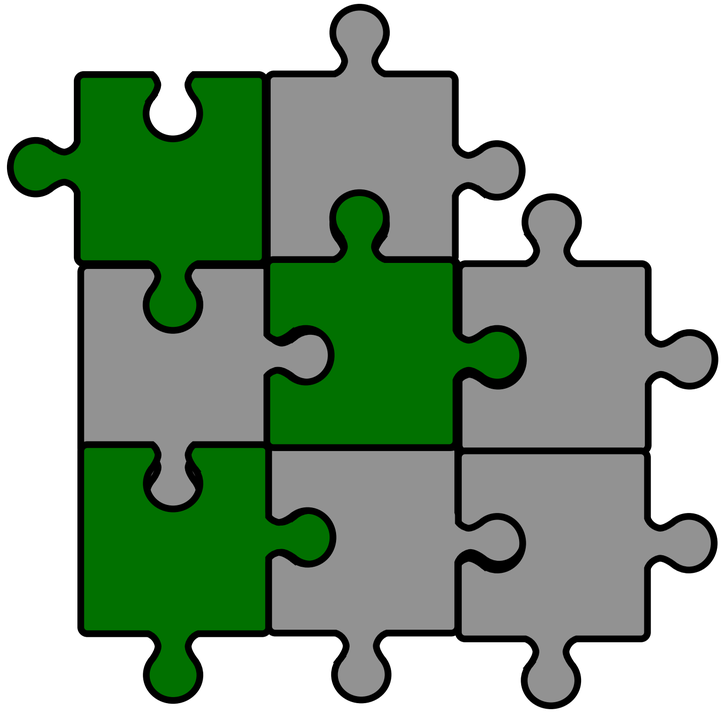
SLIM

repeat



SLIM

until a global TO is reached,
or no improvement possible



local parts not necessarily disjoint

Main difference between SLIM and LNS is that the local part is highly structured

SLIM Showcases

Problem	Local Solver	Paper
Branch-width	SAT	Lodha, Ordyniak, Sz. (SAT'17, ToCL'19)
Tree-width	SAT	Fichte, Lodha, Sz. (SAT'17)
Tree-depth	MaxSAT	Peruvemba Ramaswamy, Sz. (CP'20)
BN Structure Learning	MaxSAT	Peruvemba Ramaswamy, Sz. (AAAI'21, NeurIPS'21, UAI'22)
Decision Trees	SAT	Schidler, Sz. (AAAI'21, JAIR'24)
Graph Coloring	SAT	Schidler, Sz. (JEA'22)
Circuit Minimisation	QBF/SAT	Reichl, Slivovsky, Sz. (AAAI'23, SAT'24)

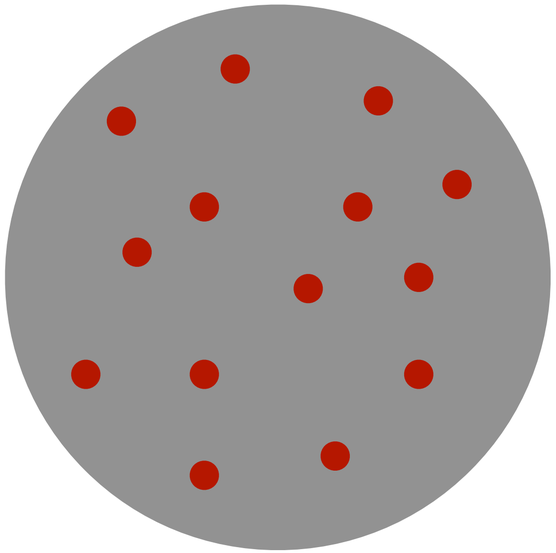
MaxSAT	MaxSAT	This paper

Maximum Satisfiability (MaxSAT) Solvers

- indispensable tools with an expansive range of applications in combinatorial optimization
- **Instance**: set of Boolean hard clauses + set of soft clauses + weight function for soft clauses
- **Task**: find an assignment to the variables that satisfies all hard clauses and maximizes the sum of weights of satisfied soft clauses (or equivalently minimizes the sum of weights of non-satisfied soft clauses)
- What we consider here is **partial weighted MaxSAT**; special case: **unweighted** instances, where all soft clauses have unit weight 1.
- **Exact solvers:** aim at finding an optimal solution
- **Anytime solvers:** aim at providing good solutions within a given time bound

MaxSAT-LNS

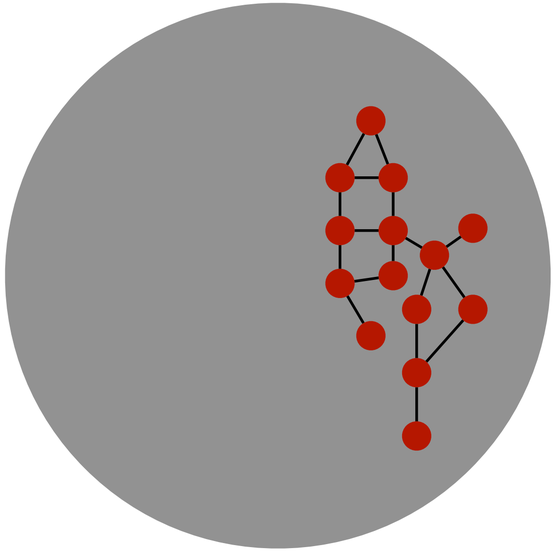
[Hickey+Bacchus 2022]



- Find a suboptimal solution with an anytime solver
- the repeatedly select random subset of fixed variables and use exact solver to improve

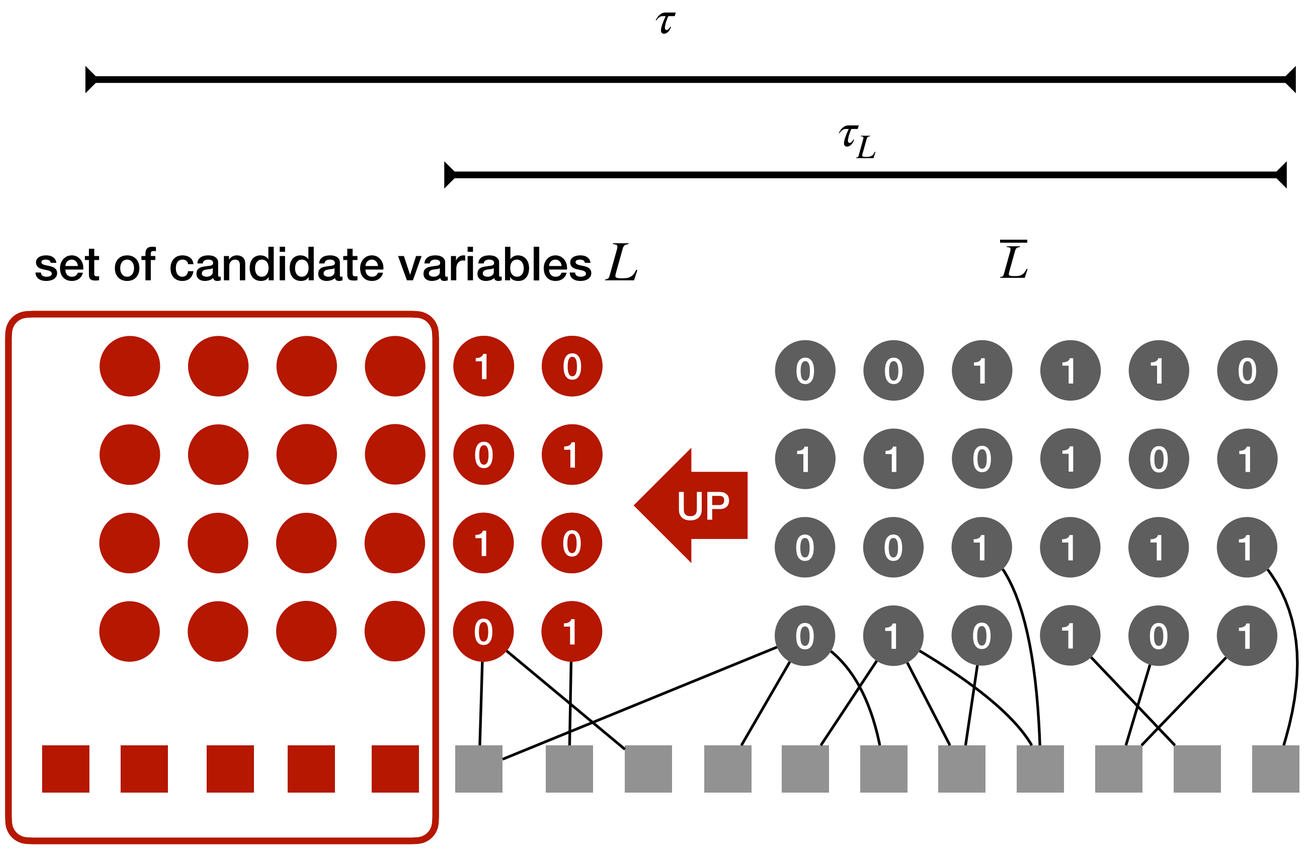
MaxSLIM

this paper



- Find a suboptimal solution with an anytime solver
- the repeatedly select subset of variables **based on the graphical structure of the instance** and use an exact solver to improve

Local Instances



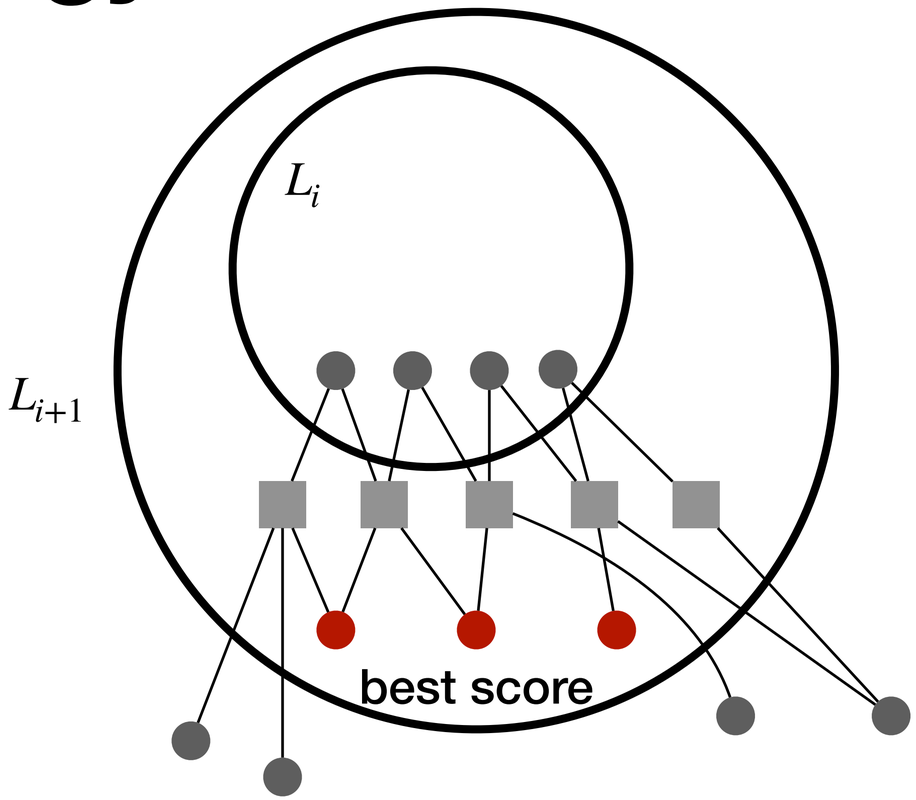
local instance

over free variables

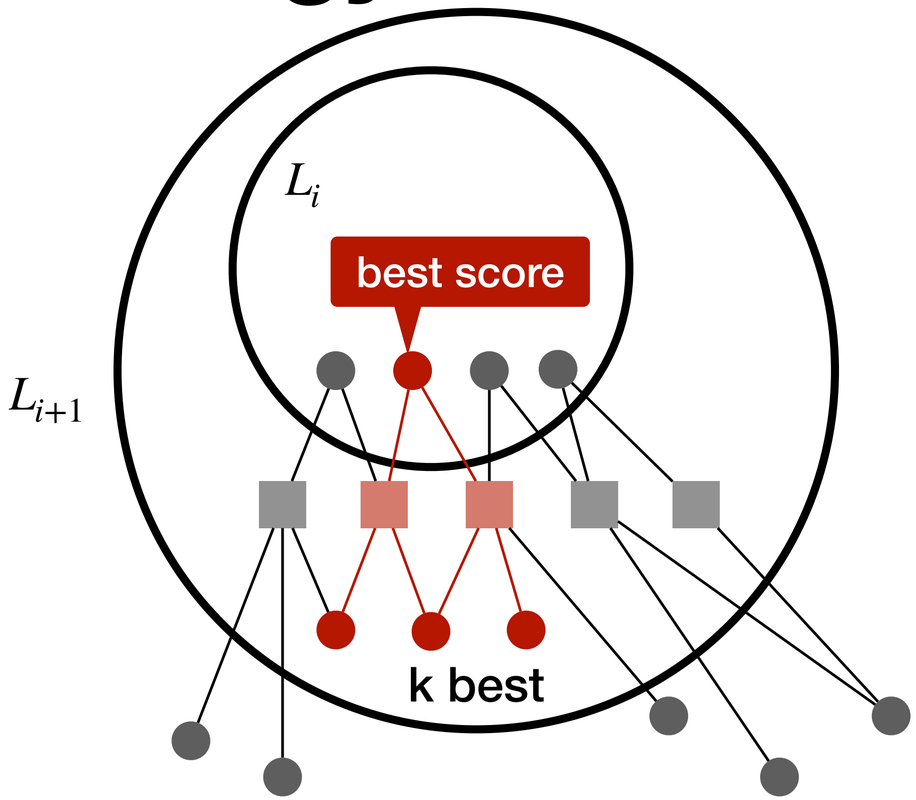
Local Instance Selection

- use an the annotated version of the incidence graph (bipartite graph between variables and clauses)
- start with $L_0 = \{v\}$ and extend $L_0 \subset L_1 \subset \ldots \subset L_i$ while free vars $\leq b$
- We propose several
  - **selection strategies** based on
  - **several metrics** try to identify variables and soft clauses that have a high probability of contributing to an improvement, using a **score** function s

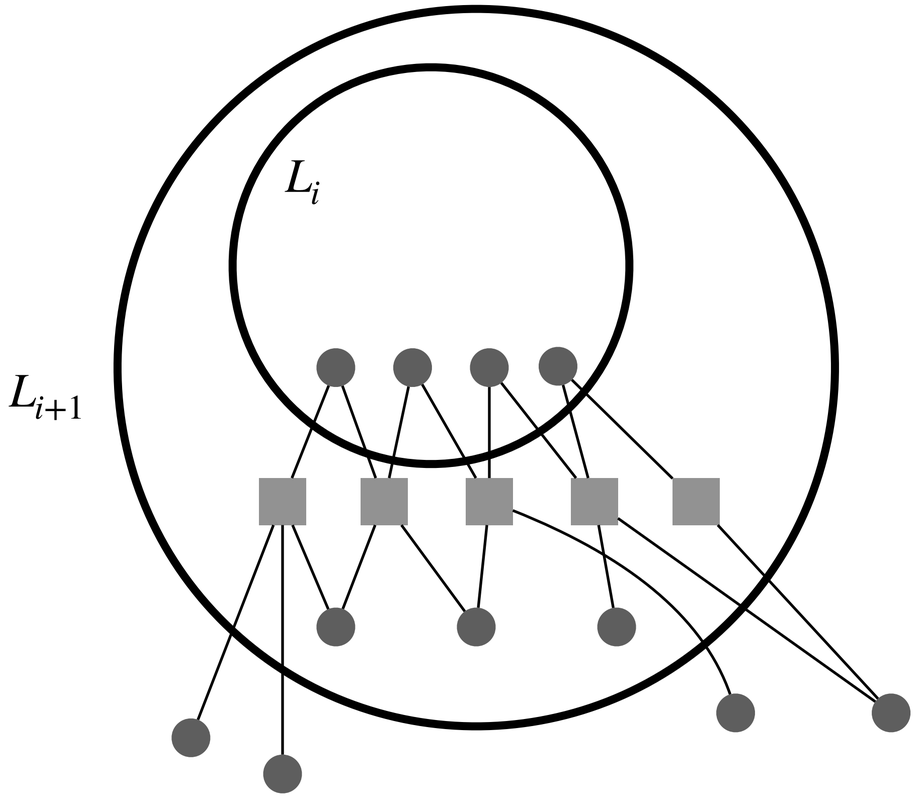
Variable Strategy



k-Adjacency Strategy



Fast Strategy



no metric

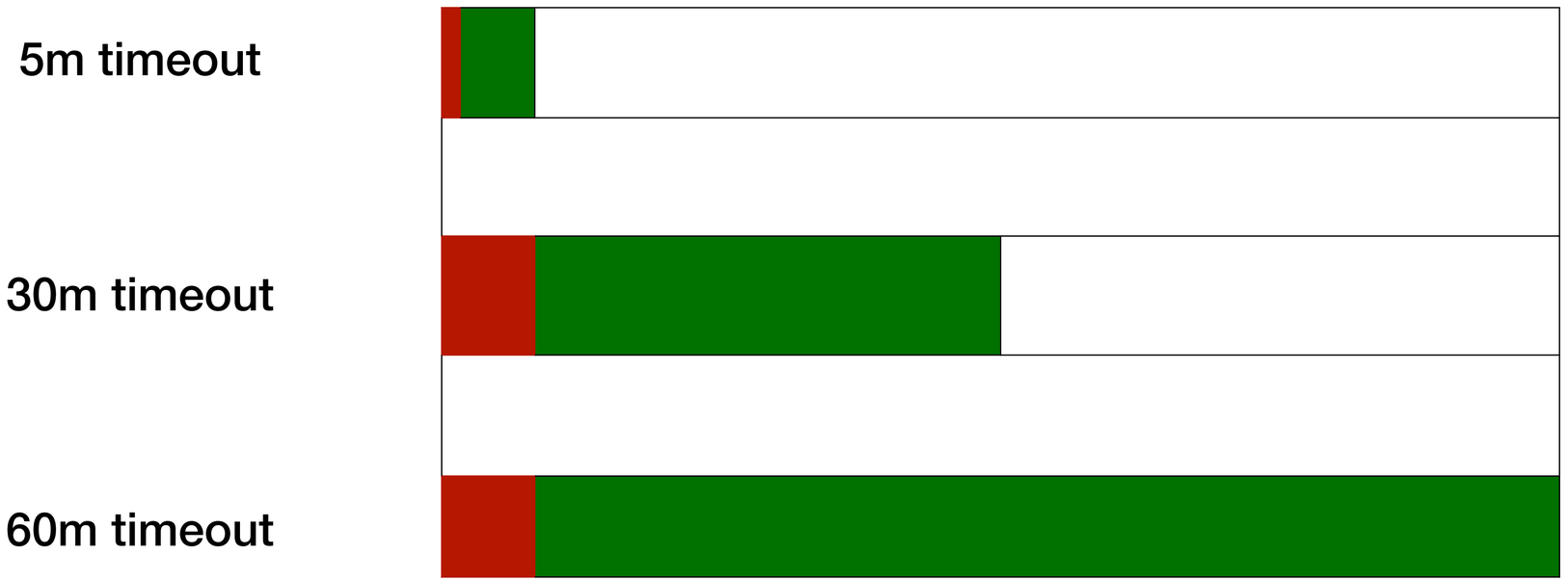
Metrics

- **Unit Metric:** score$(v) = (-1)$ number of clauses that become unsatisfied when value of v gets flipped (multiply by weight for soft clauses).
- **Satisfying Metric:** score$(v) =$ is small if v is in an unsatisfied soft clause, but value of v cannot be flipped due to a hard clause.
- **NuWLS Metric:** uses the initial weighting scheme for soft clauses of the NuWLS solver. score$(v) =$ sum of weights of all unsatisfied soft clauses v appears in that are do not get unsatisfied if value of v gets flipped.

Configuration and Instances

- **Global solver:** NuWLS-c (winner of the MaxSAT Evaluation 2023)
- **Local solver:** own core-guided solver based on the OLL algorithm
- **Local timeout:** 55 seconds (local selection plus solving within a minute)
- **Budget:** initial $b := n/10,$ every five 10 consecutive failures of improvement $b \mathrel{+}= n/10$
- **Default:** Variable Strategy with Unit Metric for unweighted, NuWLS Metric for weighted instances.
- **Instances:** 2023 MaxSAT Evaluation's anytime track (179 unweighted and 160 weighted)
- **Seeds:** three runs per solver and configuration
- **Scoring system:** as in MaxSAT Evaluation (best solver gets score 1, other solver gets (best+1)/(solver+1)

Timing



initial solution with NuWLS-c

Improvement with MaxSLIM or MaxSAT-LNS

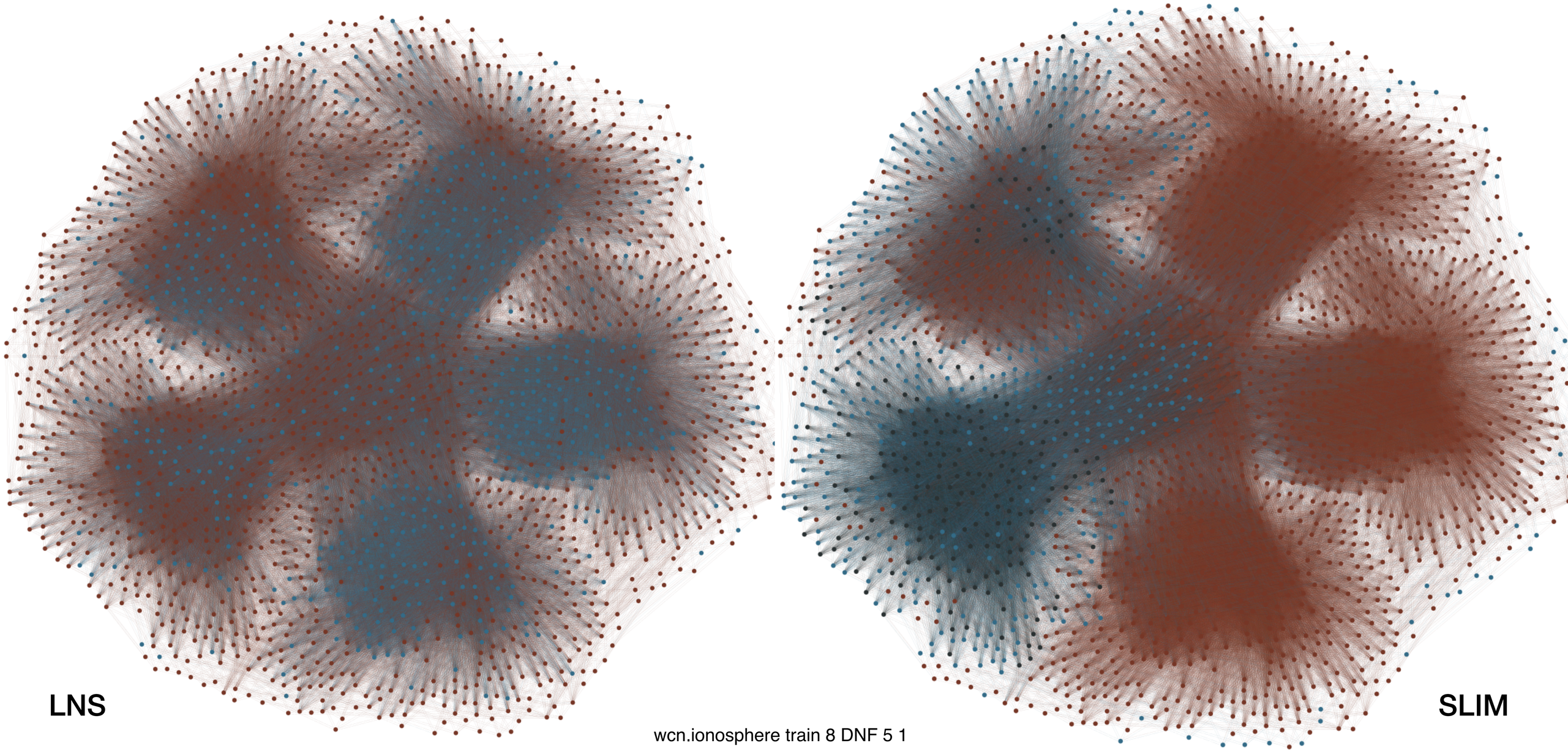
we disregard all instances where no initial solution could be found

Q1: MaxSLIM vs MaxSAT-LNS

unweighted	MaxSLIM	MaxLNS
5min	**0.887**	0.871
30min	**0.919**	0.907
60min	**0.923**	0.912

weighted	MaxSLIM	MaxLNS
5min	**0.833**	0.818
30min	**0.911**	0.894
60min	**0.917**	0.899

blue: candidate variables; dark blue: implied by UP; light blue: free



LNS

SLIM

wcn.ionosphere train 8 DNF 5 1

Q2: MaxSLIM vs NnWLS-c

unweighted	MaxSLIM	NuWLS-c
5min	**0.887**	0.885
30min	**0.919**	0.917
60min	**0.923**	0.918

weighted	MaxSLIM	NuWLS-c
5min	0.833	**0.868**
30min	**0.911**	0.889
60min	**0.917**	0.906

Q3: MaxSLIM vs Other Solvers (unweighted)

unweighted		MaxSLIM	MaxLNS	NuWLS	TT-OpenWI	Loandra	NoSAT
5min	score	**0.887**	0.871	0.885	0.876	0.820	0.586
	best	**42**	32	33	32	**42**	15
30min	score	**0.919**	0.907	0.917	0.907	0.892	0.591
	best	39	36	30	28	**41**	13
60min	score	**0.923**	0.912	0.918	0.913	0.900	0.601
	best	**41**	39	30	29	**41**	14

Q3: MaxSLIM vs Other Solvers (weighted)

weighted		MaxSLIM	MaxLNS	NuWLS	TT-OpenWI	Loandra	NoSAT
5min	score	0.833	0.818	0.868	**0.872**	0.843	0.319
	best	29	32	31	22	**35**	0
30min	score	**0.911**	0.894	0.889	0.888	0.893	0.338
	best	**42**	34	17	17	33	0
60min	score	**0.917**	0.899	0.906	0.897	0.898	0.341
	best	**41**	35	19	20	34	0

Q4: Strategies and Metrics (unweighted)

unweighted 5min Unit Metric	score	Improved
Variable Strategy	**0.887**	**97**
5-Adjacency Strategy	**0.887**	94
Fast Strategy	0.873	79

unweighted 5min Variable Strategy	score	Improved
Unit Metric	**0.887**	**97**
NuWLS Metric	0.885	**97**
Satisfying Metric	0.883	96

Q4: Strategies and Metrics (weighted)

weighted 5min NuWLS Metric	score	Improved
Variable Strategy	**0.833**	105
5-Adjacency Strategy	0.822	**107**
Fast Strategy	0.817	99

weighted 5min Variable Strategy	score	Improved
Unit Metric	0.823	105
NuWLS Metric	**0.833**	**107**
Satisfying Metric	0.824	99

Summary of results

- utilizing the graphical structure of MaxSAT instances pays off
- in particular for longer timeouts
- we have several strategies, metrics, and parameters that allow for further tuning to particular applications
- future work: interleaving local and global solving
- requires hot start of global solver

Future work: interleaving

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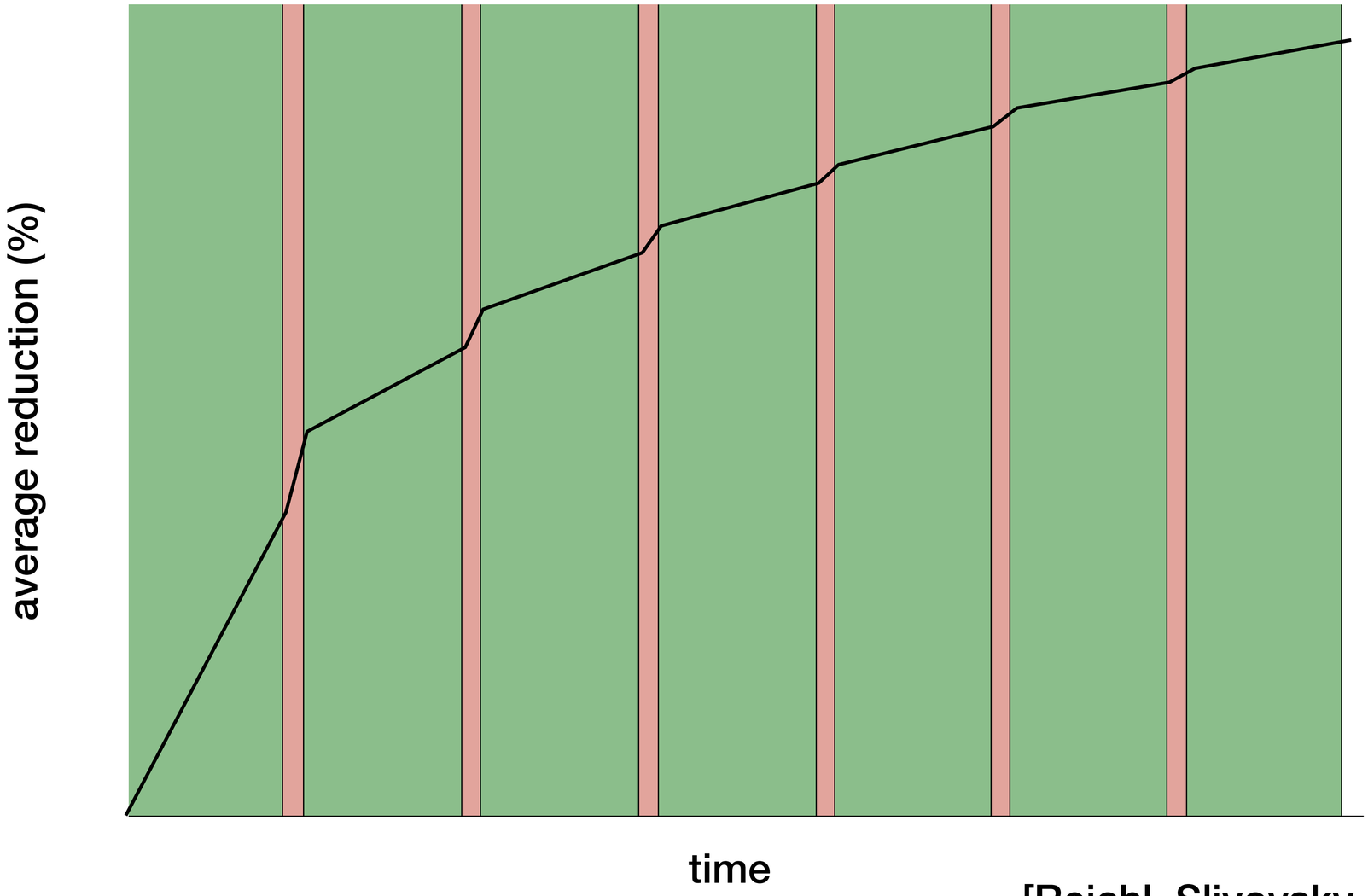
Circuit Minimization



[Reichl, Slivovsky, Sz. SAT'24]

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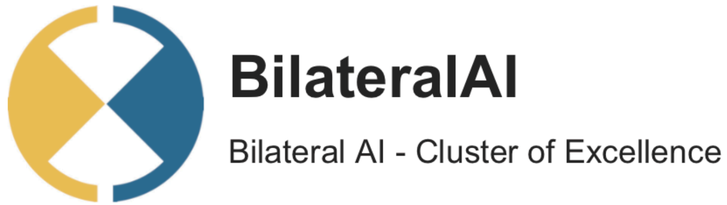

https://www.vcla.at/2024/08/open-positions-cluster-of-excellence